MIDTERM 2 FOR CALCULUS

Time: 8:15–9:55 AM, Friday, June 1, 2001

No calculator is allowed. No credit will be given for an answer without reasoning.

1.

(i) [5%] Suppose that $(0, 2)$ is a critical point of a function g with continuous second derivatives. Suppose that $g_{xx}(0, 2) = -1$, $g_{xy}(0, 2) = 2$ and $g_{yy}(0, 2) = -8$. Use second derivative test to classify the critical point $(0, 2)$.

(ii) [5%] Find an equation of the tangent plane to the surface $z = e^x \ln y$ at the point $(3, 1, 0)$.

2. [10%] Let $u = x + at$ and $v = x - at$. Then use chain rule to show that any differentiable function of the form

$$z = f(x + at) + g(x - at)$$

is a solution of the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

3. [10%] Find the directional derivative of the function $g(x, y, z) = z^3 - x^2 y$ at the point $(1, 6, 2)$ in the direction $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$.

4. [20%] Find the extreme values of the function $f(x, y) = e^{-xy}$ on the region $x^2 + 4y^2 \leq 1$.

5. [10%] Evaluate

$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x + y)\, dy, dx.$$

6. [10%] Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

7. [10%] Use triple integral to show that the volume of the solid bounded by a sphere of radius a is $\frac{4}{3} a^3 \pi$.

8. [10%] The *average value* of a function $f(x, y, z)$ over a solid region E is defined to be

$$f_{\text{ave}} = \frac{1}{V(E)} \iiint_E f(x, y, z)\, dV$$

where $V(E)$ is the volume of E. Find the average value of the function $f(x, y, z) = x + y + z$ over the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

9. [10%] Evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy$$

by reversing the order of integration.