MIDTERM 2 FOR CALCULUS

Date: 2000, June 1, 8:10–10:00AM

Each of the following problems is worth 20 points. An answer without reasoning will not be accepted.

1.

(i) [10%] Find $\partial z/\partial x$ and $\partial z/\partial y$ for $\ln(x+yz) = 1 + xy^2z^3$.

(ii) [10%] Find the curvature of the curve $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + \sin t\mathbf{k}$.

2.

(i) [10%] Compute the gradient of the function $\frac{r}{\sin r}$ where r is the distance from (x, y, z) to the origin.

(ii) [10%] Show that the function

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

does not have limit at $(0, 0)$.

3.

(i) [6%] Find the equation of the tangent plane for $z = \sin x + \sin y + \sin(x + y)$ at $(0, 0, 0)$.

(ii) [14%] Find the absolute maximum and minimum values of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disc $x^2 + y^2 \leq 4$.

4.

(i) [10%] Find the mass and center of mass of solid hemisphere of radius a (i.e., above the xy-plane and below the sphere of radius a) if the density at any point is proportional to its distance from the xy-plane.

(ii) [10%] Find the volume of the solid T enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

5.

(i) [10%] A function $f(x, y)$ is called *homogeneous of degree* n if f has continuous second-order partial derivatives and $f(tx, ty) = t^n f(x, y)$ for all t. Use the chain rule to show that if f is homogeneous of degree n, then

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y).$$

(ii) [10%] Compute $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy$.