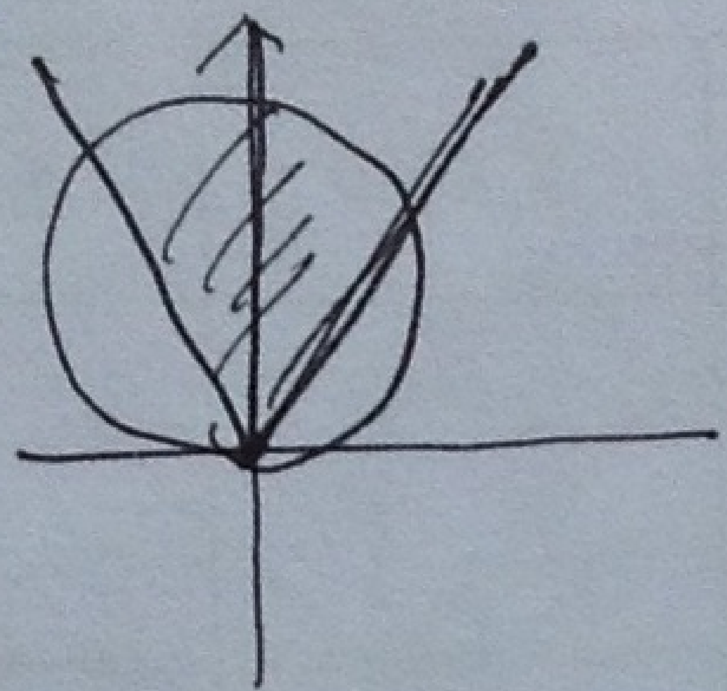
#3. Area bdd by $r = \sin\theta$, $\frac{\pi}{3} \le \theta \le \frac{2}{3}\pi$

sol:



$$2 \cdot \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2\theta \, d\theta$$

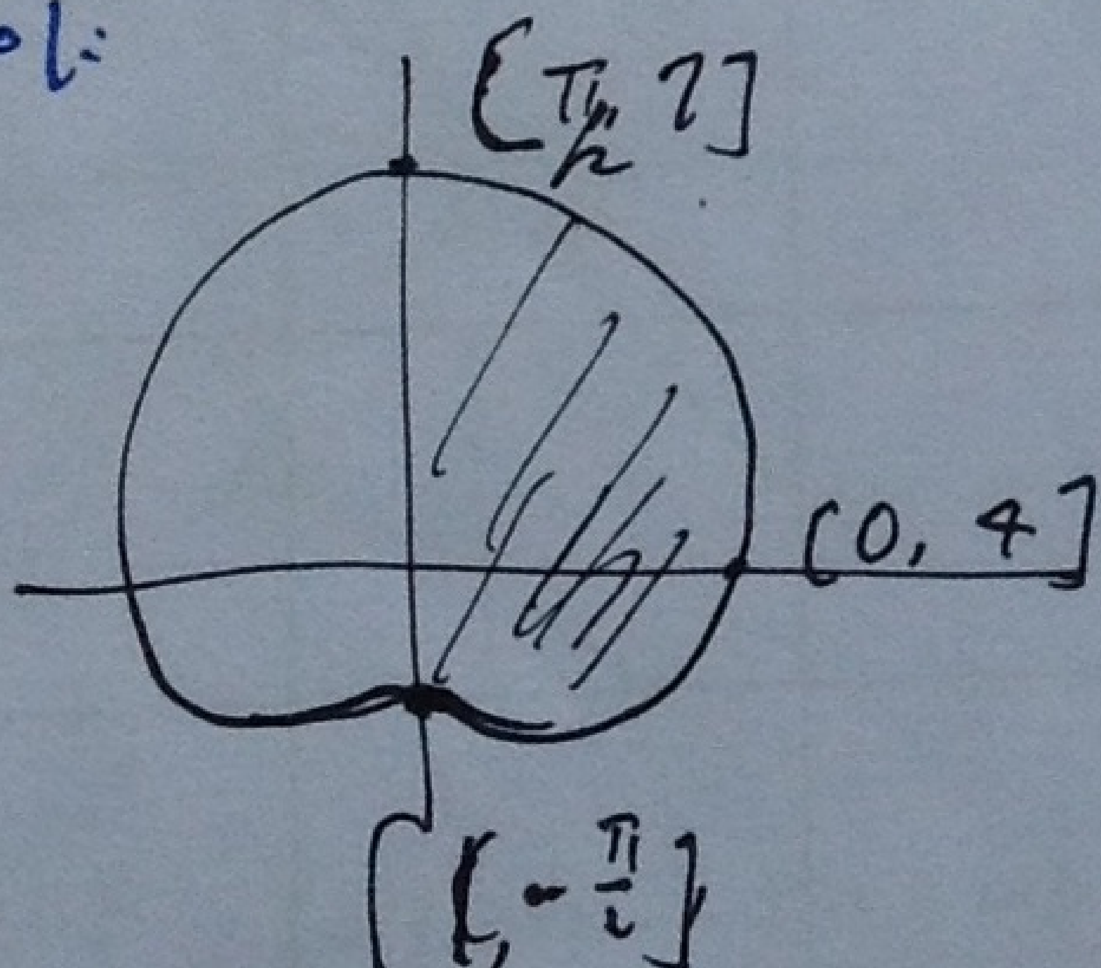
$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= \frac{1}{2}\left(\frac{\pi}{6}\right) - \frac{1}{4} \sin 2\theta \Big|_{\theta=\frac{\pi}{3}}^{\theta=\frac{\pi}{2}}$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{8}$$

#17. $r = 4 + 3\sin\theta$ Area of shaded region

sol:



$$A = \frac{1}{2} \int r^2 \, d\theta$$

$$= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 + 3\sin\theta)^2 \, d\theta$$

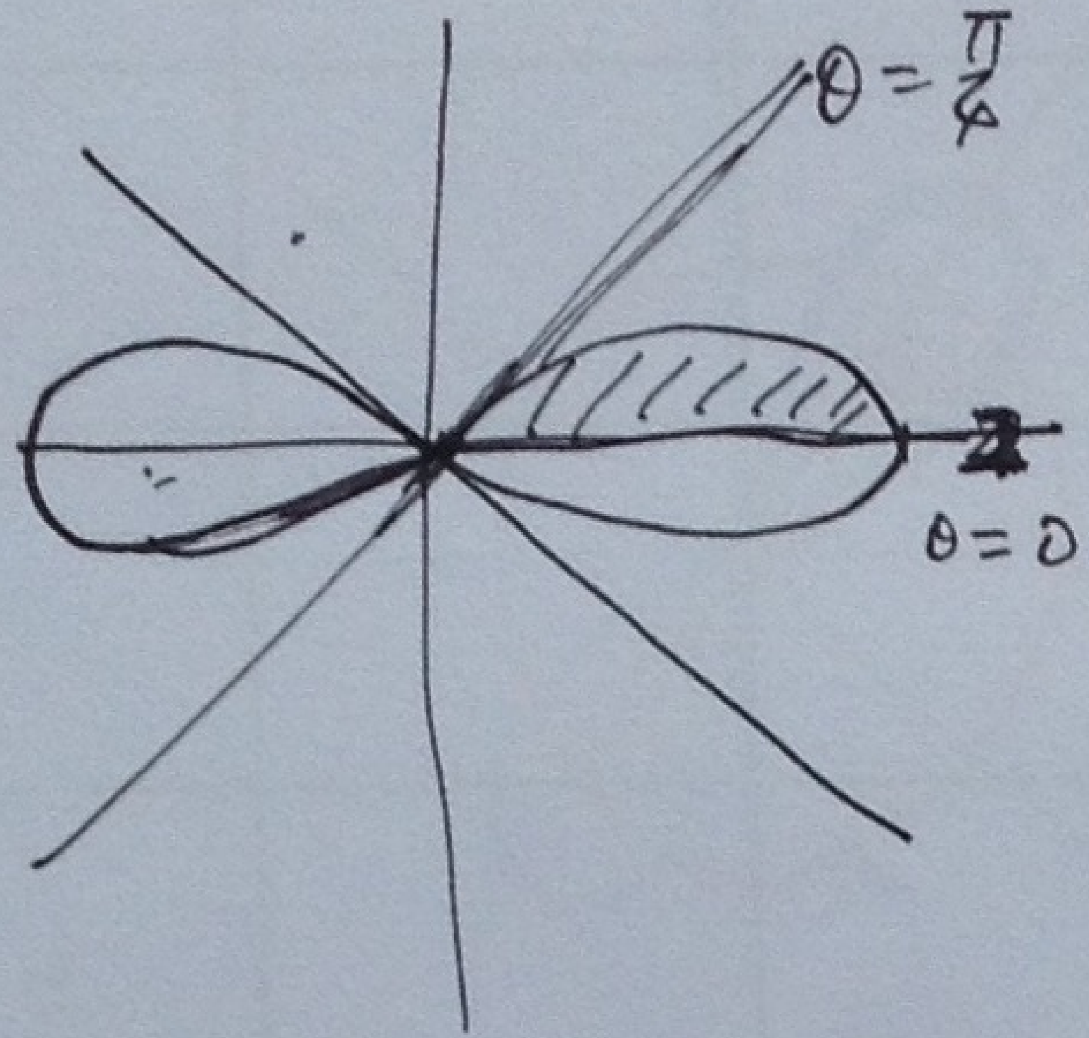
$$= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (16 + 9\sin^2\theta + 24\sin\theta) \, d\theta$$

$$= 8 \times \pi + \frac{9}{2} \cdot \frac{\pi}{2} + 12 \, \cancel{(-\cos\theta)} \Big|_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}$$

$$= \frac{41}{4}\pi$$

#9. Sketch and find area that it encloses.

sol: $r^2 = 4\cos 2\theta$



$$A = 4\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}} \overset{r^2}{\overset{\parallel}{(4\cos 2\theta)}}\, d\theta$$

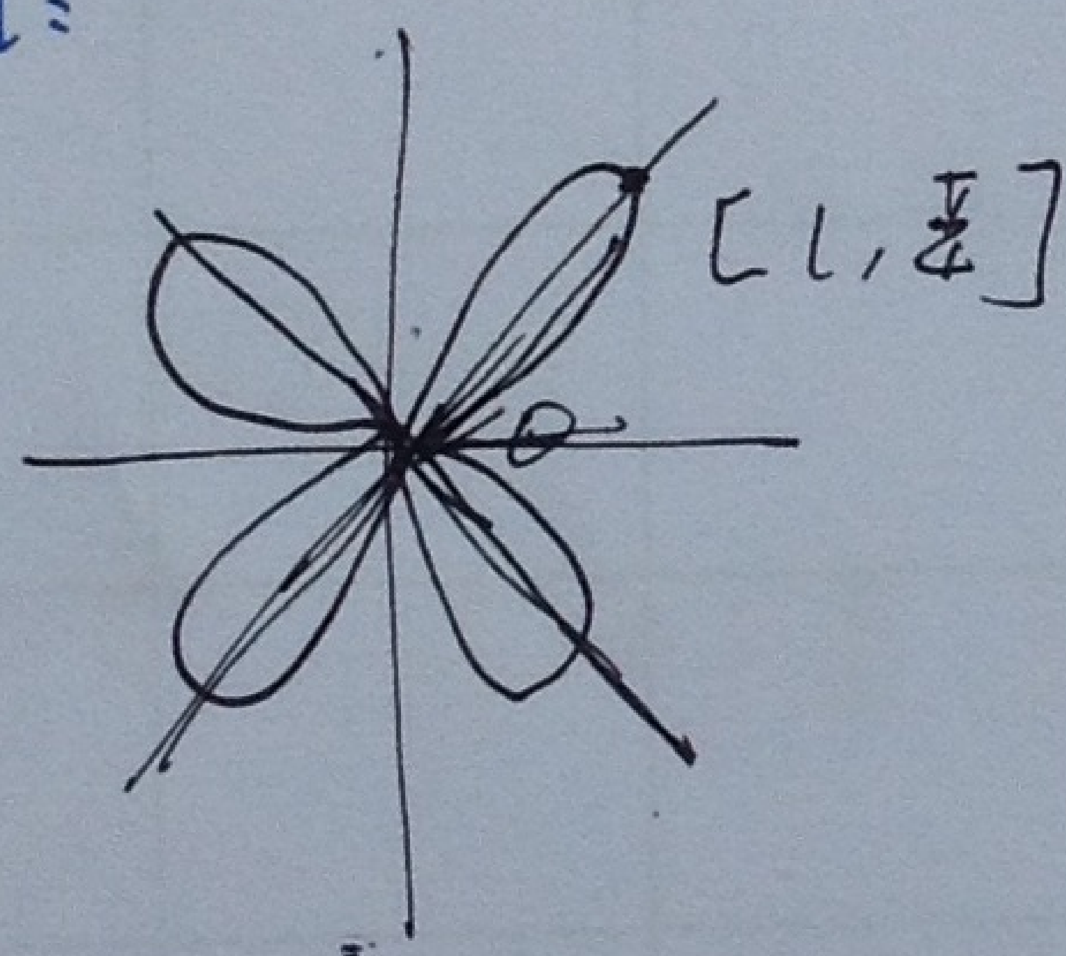
$$= 4\int_0^{\frac{\pi}{4}} \cos 2\theta \, d2\theta$$

$$= 4\cdot \sin 2\theta \Big|_{\theta=0}^{\theta=\frac{\pi}{4}}$$

$$= \underline{4} \ \#$$

#15 One loop 之 area of $r = \sin 2\theta$

sol:



$$A = 2\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}} \sin^2 2\theta \, d\theta$$

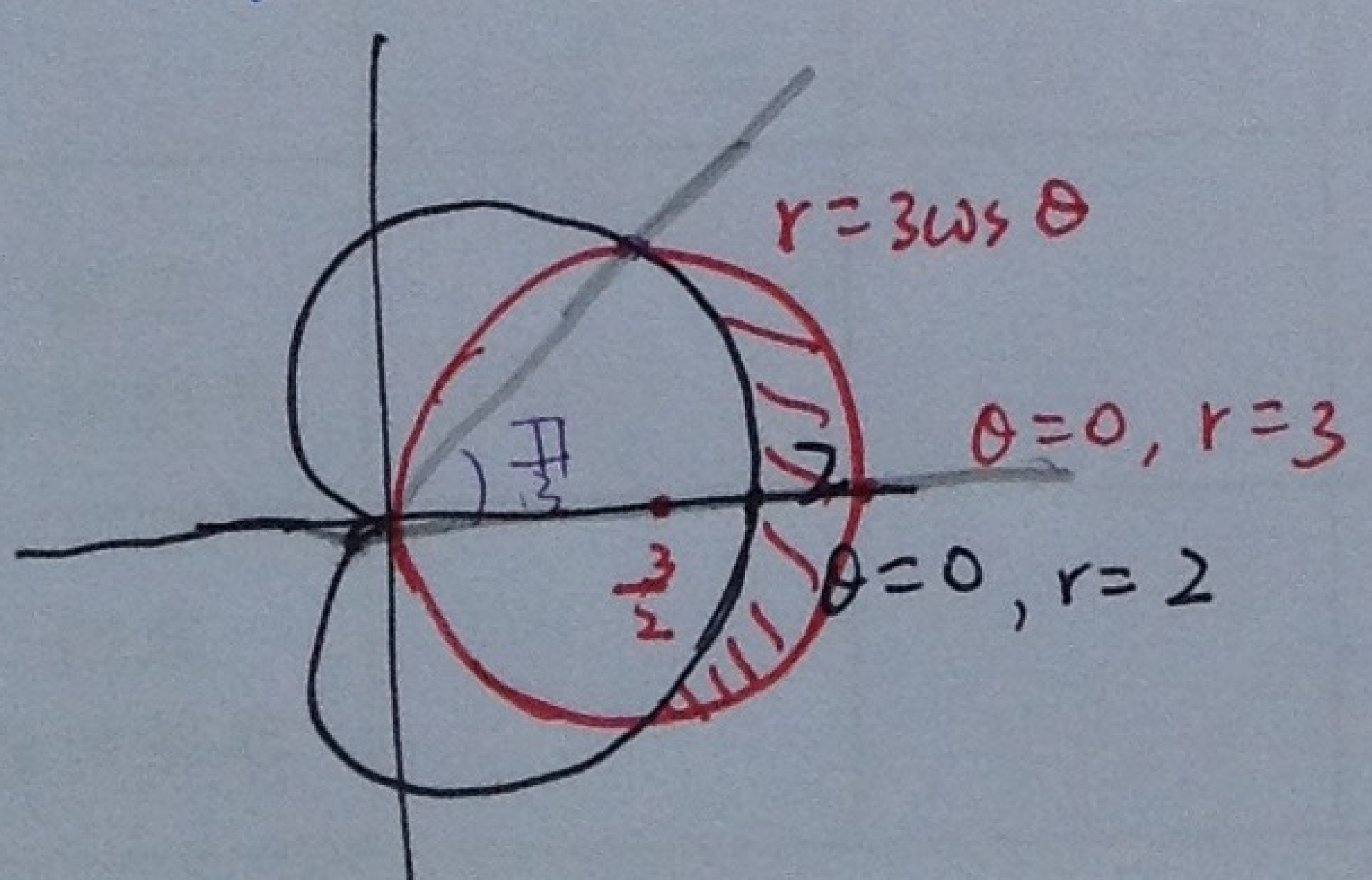
令 $2\theta = u$ $\quad d\theta = \frac{1}{2}du$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin^2 u \, du$$

$$= \frac{1}{2}\cdot\frac{\pi}{2}\times\frac{1}{2} = \underline{\frac{\pi}{8}} \ \#$$

#21 Inside $r = 3\cos\theta$ and Outside $r = 1+\cos\theta$ 之 area

sol:



交点 $3\cos\theta = 1+\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}$

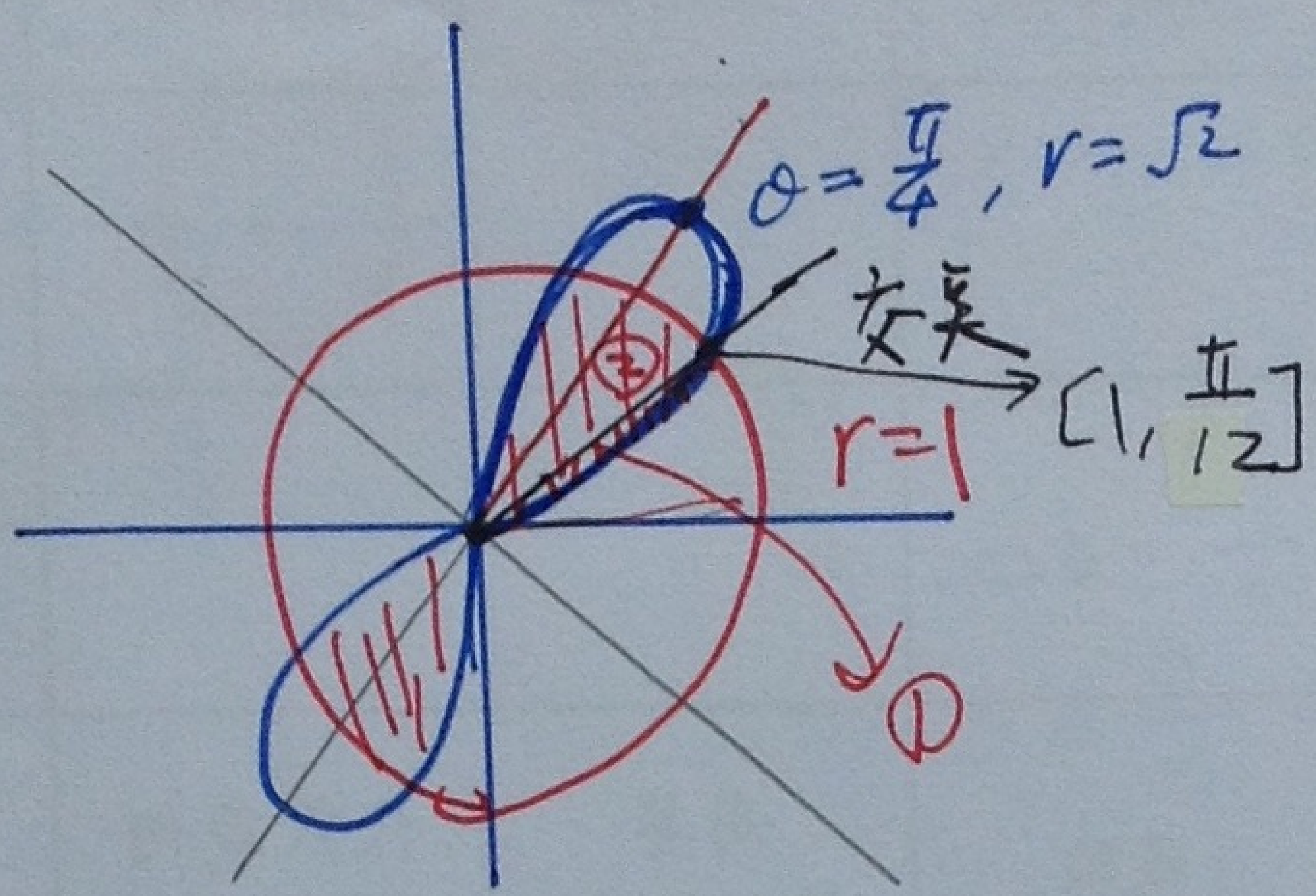
$\theta = \frac{\pi}{3}$

$$A = 2\cdot\frac{1}{2}\int_0^{\frac{\pi}{3}} \left[(3\cos\theta)^2 - (1+\cos\theta)^2\right] d\theta$$

$$= \int_0^{\frac{\pi}{3}} \underset{3+4\cos 2\theta - 2\cos\theta}{\underset{\parallel}{8\cos^2\theta - 2\cos\theta - 1}} \, d\theta = \underline{\pi} \ \#$$

※. #26. Inside $r^2 = 2\sin 2\theta \underset{\text{and}}{\wedge} r=1$ 之區域 area

sol:



解交點

$2\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{12}$

① : $0 \le \theta \le \frac{\pi}{12}$ $r^2 = 2\sin 2\theta$

$$\text{Area}(①) = \frac{1}{2}\int_0^{\frac{\pi}{12}} 2\sin 2\theta \, d\theta$$

$$= \frac{-1}{2}\cos 2\theta \Big|_{\theta=0}^{\theta=\frac{\pi}{12}} = \frac{2-\sqrt{3}}{4}.$$

② : $\frac{\pi}{12} \le \theta \le \frac{\pi}{4}$, $r=1$

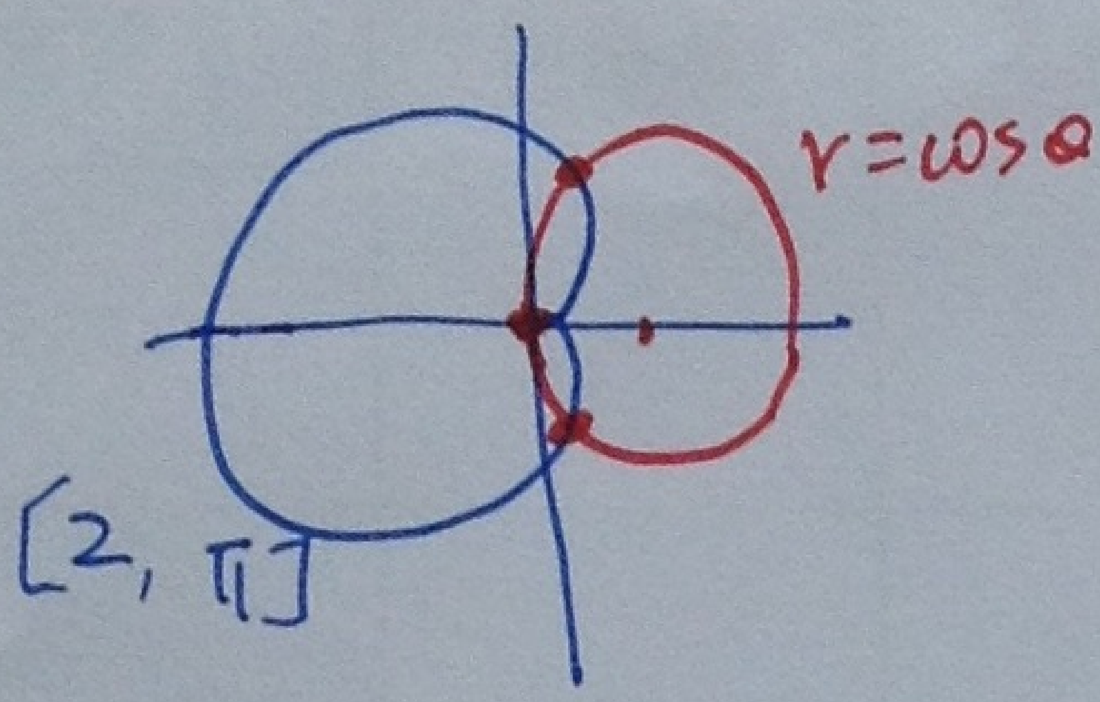
$$\text{Area}(②) = \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 1 \, d\theta = \frac{1}{12}\pi$$

所求 $= 4(① + ②) = 4\left[\frac{2-\sqrt{3}}{4} + \frac{1}{12}\pi\right] = \underline{2-\sqrt{3}+\frac{\pi}{3}}$ #

§9.4 P.4

#29. 求交点 $r=\cos\theta$, $r=1-\cos\theta$

sol: 一定要画图 (3个交点)



$$\begin{cases} r=\cos\theta \\ r=1-\cos\theta \end{cases}$$

$\Rightarrow 2\cos\theta=1$

$\Rightarrow \cos\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{3}$ or $\frac{5}{3}\pi$

$\downarrow \; r=\frac{1}{2}$ $\quad \downarrow \; r=\frac{1}{2}$

又 $[0,0]$ 满足 $r=1-\cos\theta$

$\parallel$

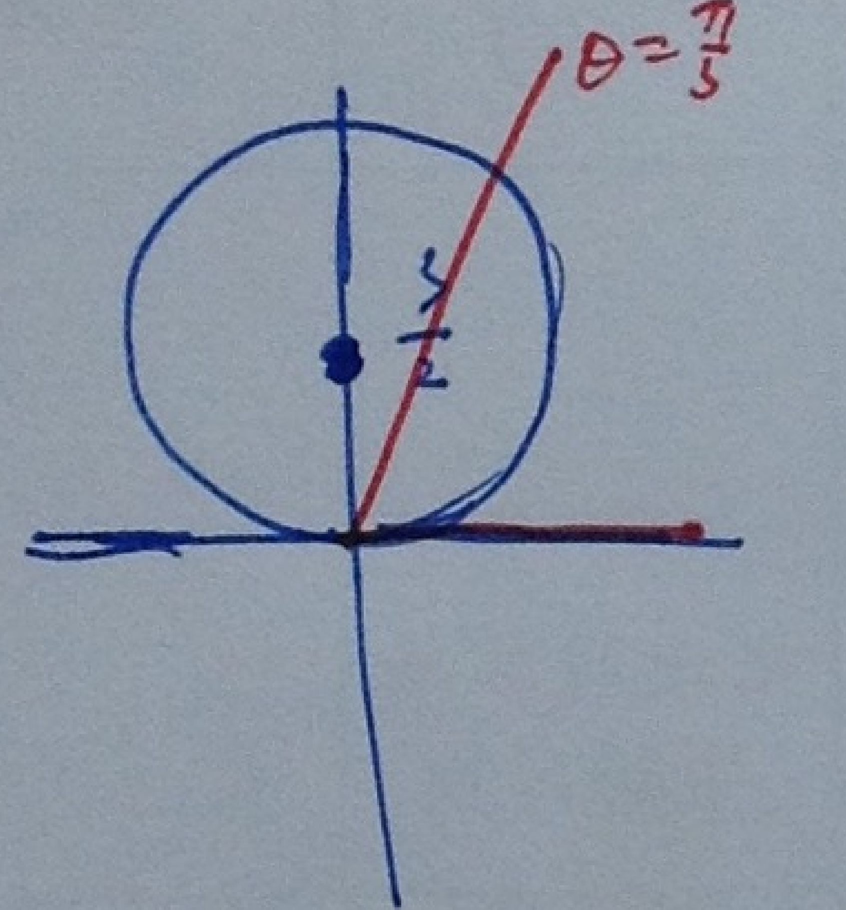
$[0,\frac{\pi}{2}]$ 满足 $r=\cos\theta$

∴ 共有3个交点

$[0,0]$, $[\frac{1}{2},\frac{\pi}{3}]$,

$[\frac{1}{2},\frac{5}{3}\pi]$ #

#33 length of $r=3\sin\theta$, $0\le\theta\le\frac{\pi}{3}$



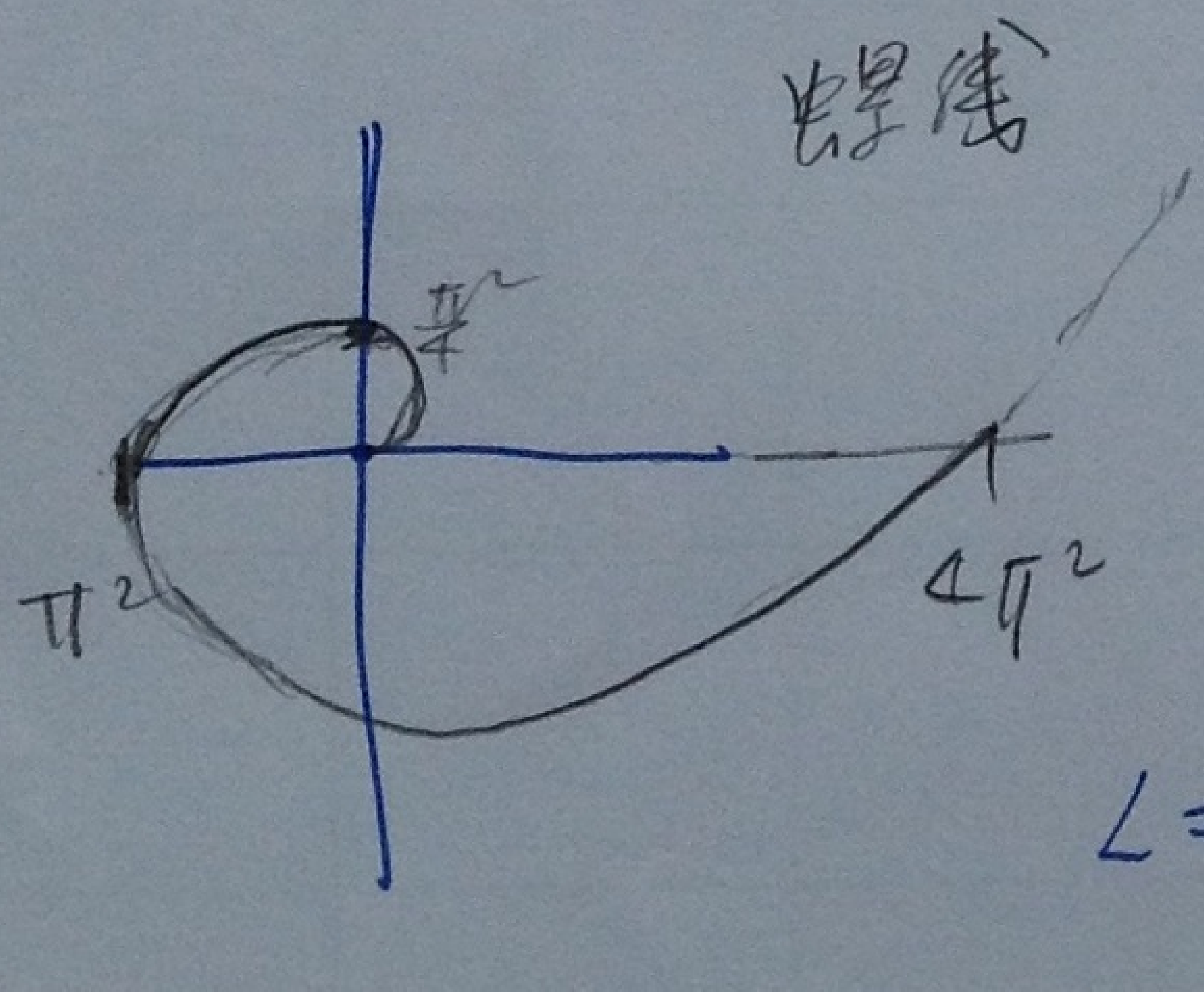
$ds=\sqrt{r^2+\dot{r}^2}\,d\theta$

$=3\sqrt{\sin^2\theta+\cos^2\theta}\,d\theta$

$=3\,d\theta$

$L=\int ds=3\int_0^{\frac{\pi}{3}}d\theta=\pi$ #

#35 length of $r=\theta^2$, $0\le\theta\le2\pi$



$ds=\sqrt{r^2+\dot{r}^2}\,d\theta$

$=\sqrt{\theta^4+4\theta^2}\,d\theta$

$=\theta\sqrt{\theta^2+4}\,d\theta$

$L=\int ds=\int_0^{2\pi}\theta\sqrt{\theta^2+4}\,d\theta$

$=\frac{1}{2}\int_0^{2\pi}(\theta^2+4)^{\frac{1}{2}}\,d(\theta^2+4)$

$=\frac{1}{2}\,\frac{2}{3}\,(\theta^2+4)^{\frac{3}{2}}\Big|_{\theta=0}^{\theta=2\pi}=\frac{8}{3}(\pi^2+1)^{\frac{3}{2}}-\frac{8}{3}$ #