§11.2

#3. $\lim_{(x,y)\to(5,-2)} (x^5+4x^3y-5xy^2) = [5^5+4\cdot5^3\cdot(-2)-5\times5\times(-2)^2]$

因 poly. 為 contr.

故 $x=5, y=-2$ 代入

即為 limit

$= 2025$ ※

#5. $\lim_{(x,y)\to(0,0)} \boxed{\dfrac{y^4}{x^4+3y^4}} := f(x,y)$ DNE

Since approach $(0,0)$ along the x-axis. Then $f(x,0)=0$, $x\neq 0$

so $f(x,y)\to 0$ along the x-axis. as $x\to 0$

Now approach $(0,0)$ along the y-axis. Then $f(0,y)=\frac{1}{3}$, $y\neq 0$

so $f(x,y)\to\frac{1}{3}$. Since f has two different limits along two different lines, the limit does not exist. ※

#7. $\lim_{(x,y)\to(0,0)} \boxed{\dfrac{xy\cos y}{3x^2+y^2}} := f(x,y)$

$f\big|_{x\text{-axis}}(x,y) = f(x,0) = 0 \to 0$ as $x\to 0$

$f\big|_{y=x}(x,y) = f(x,x) = \dfrac{x^2\cos x}{4x^2} = \frac{1}{4}\cos x \to \frac{1}{4}$ as $x\to 0$

Hence, the limit DNE ※

#9. $\lim_{(x,y)\to(0,0)} \boxed{\dfrac{xy}{\sqrt{x^2+y^2}}} := f(x,y)$

$|y| \leq \sqrt{x^2+y^2}$

$\Rightarrow |x||y| \leq |x|\sqrt{x^2+y^2}$

$\Rightarrow \underset{\downarrow \atop 0}{0} \leq |f(x,y)| \leq \underset{\downarrow \atop 0}{|x|}$ as $(x,y)\to(0,0)$

By Squeeze Thm, $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ ※

§11.2

#15. $\lim\limits_{(x,y,z)\to(0,0,0)} \boxed{\dfrac{xy+yz^2+xz^2}{x^2+y^2+z^2}} := f(x,y,z)$

along x-axis. $f(x,0,0) = \dfrac{0}{x^2} = 0 \to 0$ as $x \to 0$

along $y = x$ $\quad f(x,x,0) = \dfrac{x^2}{2x^2} = \dfrac{1}{2} \to \dfrac{1}{2}$ as $x \to 0$

Thus the limit DNE. #

#19. $g(t) = t^2 + \sqrt{t}$, $f(x,y) = 2x+3y-6$.

Find the set on which $h(x,y) = g(f(x,y))$ is contn.

$$(x,y) \xrightarrow[\text{is contn. on } \mathbb{R}^2]{f} f(x,y) = t \xrightarrow[\text{is contn. for } t \ge 0]{g} g(t) = z$$

(有定义即連續)

$h = g \circ f$ is contn 在那些 (x,y) s.t

$f(x,y) = 2x+3y-6 \ge 0$

h is contn on its domain $D = \{(x,y) \mid y \ge -\frac{2}{3}x + 2\}$ #

#23 $G(x,y) = \ln(x^2+y^2-4)$ is contn on $\{(x,y) \mid x^2+y^2 > 4\}$ #

#25. $f(x,y,z) = \dfrac{\sqrt{y}}{x^2-y^2+z^2}$ is contn. on $\{(x,y,z) \mid y \ge 0,\ y \ne \sqrt{x^2+z^2}\}$ #

有定义處 $x^2+z^2 \ne y^2$ 且 $y \ge 0$

#27.
$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2}, & (x,y) \ne (0,0) \\ 1, & (0,0) \end{cases}$$

is contn on $\mathbb{R}^2 - \{(0,0)\}$ #

因 $0 \le \left|\dfrac{x^2y^3}{2x^2+y^2}\right| \le \dfrac{(2x^2+y^2)|y^3|}{2x^2+y^2} = |y|^3$

$\downarrow \qquad\qquad \downarrow$

$0 \qquad\qquad 0$ as $(x,y)\to(0,0)$

故 $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 \ne f(0,0)$

#29. Use polar coordinate to find the limit

$$0 \le \left|\frac{x^3+y^3}{x^2+y^2}\right| = \left|\frac{r^3(\cos^3\theta+\sin^3\theta)}{r^2}\right| = r\left|\cos^3\theta+\sin^3\theta\right| \le 2r = 2\sqrt{x^2+y^2}$$

$\downarrow \qquad\qquad\qquad \to 0$

0 $\quad \therefore \lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2} = 0$ by Squeeze Thm