9. For simplicity, consider a unit cube positioned with its back left corner at the origin. Vector representations of the diagonals joining the points $(0,0,0)$ to $(1,1,1)$ and $(1,0,0)$ to $(0,1,1)$ are $\langle 1,1,1\rangle$ and $\langle -1,1,1\rangle$. Let θ be the angle between these two vectors. $\langle 1,1,1\rangle \cdot \langle -1,1,1\rangle = -1+1+1 = 1 = |\langle 1,1,1\rangle|\,|\langle -1,1,1\rangle|\cos\theta = 3\cos\theta \quad\Rightarrow\quad \cos\theta = \frac{1}{3} \quad\Rightarrow$ $\theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 71^\circ$.

10. $\overrightarrow{AB} = \langle 1,3,-1\rangle$, $\overrightarrow{AC} = \langle -2,1,3\rangle$ and $\overrightarrow{AD} = \langle -1,3,1\rangle$. By Equation 10.4.10,

$$\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right) = \begin{vmatrix} 1 & 3 & -1 \\ -2 & 1 & 3 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} - 3\begin{vmatrix} -2 & 3 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} = -8-3+5 = -6.$$

The volume is $\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right| = 6$ cubic units.

11. $\overrightarrow{AB} = \langle 1,0,-1\rangle$, $\overrightarrow{AC} = \langle 0,4,3\rangle$, so

(a) a vector perpendicular to the plane is $\overrightarrow{AB}\times\overrightarrow{AC} = \langle 0+4, -(3+0), 4-0\rangle = \langle 4,-3,4\rangle$.

(b) $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\sqrt{16+9+16} = \frac{\sqrt{41}}{2}$.

12. $\mathbf{D} = 4\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k}$, $\quad W = \mathbf{F}\cdot\mathbf{D} = 12+15+60 = 87$ J

13. Let F_1 be the magnitude of the force directed 20° away from the direction of shore, and let F_2 be the magnitude of the other force. Separating these forces into components parallel to the direction of the resultant force and perpendicular to it gives $F_1\cos 20^\circ + F_2\cos 30^\circ = 255$ **(1)**, and $F_1\sin 20^\circ - F_2\sin 30^\circ = 0 \quad\Rightarrow\quad F_1 = F_2\dfrac{\sin 30^\circ}{\sin 20^\circ}$ **(2)**. Substituting **(2)** into **(1)** gives $F_2(\sin 30^\circ\cot 20^\circ + \cos 30^\circ) = 255 \quad\Rightarrow\quad F_2 \approx 114$ N. Substituting this into **(2)** gives $F_1 \approx 166$ N.

14. $|\boldsymbol{\tau}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = (0.40)(50)\sin(90^\circ - 30^\circ) \approx 17.3$ N·m.

15. The line has direction $\mathbf{v} = \langle -3,2,3\rangle$. Letting $P_0 = (4,-1,2)$, parametric equations are $x = 4-3t$, $y = -1+2t$, $z = 2+3t$.

16. A direction vector for the line is $\mathbf{v} = \langle 3,2,1\rangle$, so parametric equations for the line are $x = 1+3t$, $y = 2t$, $z = -1+t$.

17. A direction vector for the line is a normal vector for the plane, $\mathbf{n} = \langle 2,-1,5\rangle$, and parametric equations for the line are $x = -2+2t$, $y = 2-t$, $z = 4+5t$.

18. Since the two planes are parallel, they will have the same normal vectors. Then we can take $\mathbf{n} = \langle 1,4,-3\rangle$ and an equation of the plane is $1(x-2) + 4(y-1) - 3(z-0) = 0$ or $x + 4y - 3z = 6$.

19. Here the vectors $\mathbf{a} = \langle 4-3, 0-(-1), 2-1\rangle = \langle 1,1,1\rangle$ and $\mathbf{b} = \langle 6-3, 3-(-1), 1-1\rangle = \langle 3,4,0\rangle$ lie in the plane, so $\mathbf{n} = \mathbf{a}\times\mathbf{b} = \langle -4,3,1\rangle$ is a normal vector to the plane and an equation of the plane is $-4(x-3) + 3(y-(-1)) + 1(z-1) = 0$ or $-4x + 3y + z = -14$.

20. If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle 2,-1,3\rangle$ is one vector in the plane. We can verify that the given point $(1,2,-2)$ does not lie on this line. The point $(0,3,1)$ is on the line (obtained by putting $t=0$) and hence in the plane, so the vector $\mathbf{b} = \langle 0-1, 3-2, 1-(-2)\rangle = \langle -1,1,3\rangle$ lies in the plane, and a normal vector is $\mathbf{n} = \mathbf{a}\times\mathbf{b} = \langle -6,-9,1\rangle$. Thus an equation of the plane is $-6(x-1) - 9(y-2) + (z+2) = 0$ or $6x + 9y - z = 26$.

21. Substitution of the parametric equations into the equation of the plane gives $2x - y + z = 2(2-t) - (1+3t) + 4t = 2 \Rightarrow -t + 3 = 2 \Rightarrow t = 1$. When $t = 1$, the parametric equations give $x = 2 - 1 = 1$, $y = 1 + 3 = 4$ and $z = 4$. Therefore, the point of intersection is $(1, 4, 4)$.

22. Use the formula proven in Exercise 10.4.39(a). In the notation used in that exercise, $\mathbf{a}$ is just the direction of the line; that is, $\mathbf{a} = \langle 1, -1, 2 \rangle$. A point on the line is $(1, 2, -1)$ (setting $t = 0$), and therefore $\mathbf{b} = \langle 1-0, 2-0, -1-0 \rangle = \langle 1, 2, -1 \rangle$.

Hence $d = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \dfrac{|\langle 1,-1,2 \rangle \times \langle 1,2,-1 \rangle|}{\sqrt{1+1+4}} = \dfrac{|\langle -3,3,3 \rangle|}{\sqrt{6}} = \sqrt{\dfrac{27}{6}} = \dfrac{3}{\sqrt{2}}$.

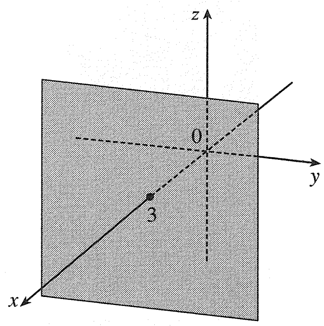
23. Since the direction vectors $\langle 2, 3, 4 \rangle$ and $\langle 6, -1, 2 \rangle$ aren't parallel, neither are the lines. For the lines to intersect, the three equations $1 + 2t = -1 + 6s$, $2 + 3t = 3 - s$, $3 + 4t = -5 + 2s$ must be satisfied simultaneously. Solving the first two equations gives $t = \frac{1}{5}$, $s = \frac{2}{5}$ and checking we see these values don't satisfy the third equation. Thus the lines aren't parallel and they don't intersect, so they must be skew.

24. (a) The normal vectors are $\langle 1, 1, -1 \rangle$ and $\langle 2, -3, 4 \rangle$. Since these vectors aren't parallel, neither are the planes parallel. Also $\langle 1, 1, -1 \rangle \cdot \langle 2, -3, 4 \rangle = 2 - 3 - 4 = -5 \neq 0$ so the normal vectors, and thus the planes, are not perpendicular.

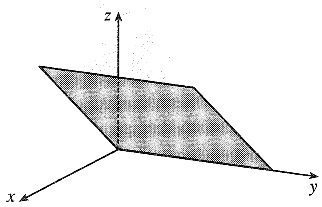
(b) $\cos\theta = \dfrac{\langle 1,1,-1 \rangle \cdot \langle 2,-3,4 \rangle}{\sqrt{3}\sqrt{29}} = -\dfrac{5}{\sqrt{87}}$ and $\theta = \cos^{-1}\left(-\frac{5}{\sqrt{87}}\right) \approx 122^\circ$ [or we can say $\approx 58^\circ$].

25. By Exercise 10.5.51, $D = \dfrac{|2 - 24|}{\sqrt{26}} = \dfrac{22}{\sqrt{26}}$.

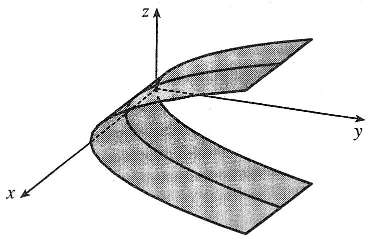
26. The equation $x = 3$ represents a plane parallel to the yz-plane and 3 units in front of it.



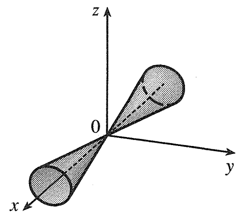
27. The equation $x = z$ represents a plane perpendicular to the xz-plane and intersecting the xz-plane in the line $x = z$, $y = 0$.



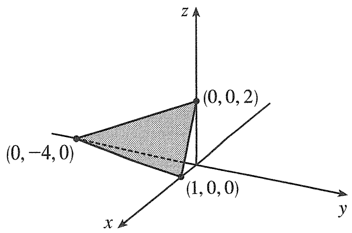
28. The equation $y = z^2$ represents a parabolic cylinder whose trace in the xz-plane is the x-axis and which opens to the right.



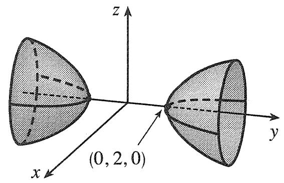
29. The equation $x^2 = y^2 + 4z^2$ represents a (right elliptical) cone with vertex at the origin and axis the x-axis.



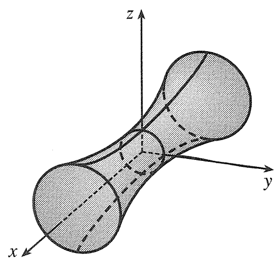
30. $4x - y + 2z = 4$ is a plane with intercepts $(1, 0, 0)$, $(0, -4, 0)$, and $(0, 0, 2)$.



31. An equivalent equation is $-x^2 + \dfrac{y^2}{4} - z^2 = 1$, a hyperboloid of two sheets with axis the y-axis. For $|y| > 2$, traces parallel to the xz-plane are circles.



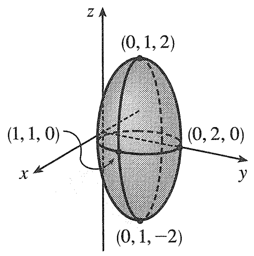
32. An equivalent equation is $-x^2 + y^2 + z^2 = 1$, a hyperboloid of one sheet with axis the x-axis.



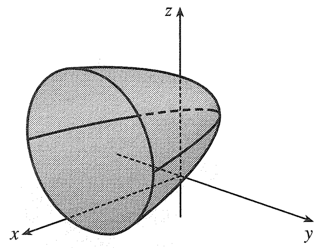
33. Completing the square in y gives

$4x^2 + 4(y-1)^2 + z^2 = 4$ or $x^2 + (y-1)^2 + \dfrac{z^2}{4} = 1$,

an ellipsoid centered at $(0, 1, 0)$.



34. Completing the square in y and z gives $x = (y-1)^2 + (z-2)^2$, a circular paraboloid with vertex $(0, 1, 2)$ and axis the horizontal line $y = 1$, $z = 2$.



35. $4x^2 + y^2 = 16 \quad \Leftrightarrow \quad \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$. The equation of the ellipsoid is $\dfrac{x^2}{4} + \dfrac{y^2}{16} + \dfrac{z^2}{c^2} = 1$, since the horizontal trace in the plane $z = 0$ must be the original ellipse. The traces of the ellipsoid in the yz-plane must be circles since the surface is obtained by rotation about the x-axis. Therefore, $c^2 = 16$ and the equation of the ellipsoid is $\dfrac{x^2}{4} + \dfrac{y^2}{16} + \dfrac{z^2}{16} = 1 \quad \Leftrightarrow$

$4x^2 + y^2 + z^2 = 16$.

36. The distance from a point $P(x, y, z)$ to the plane $y = 1$ is $|y - 1|$, so the given condition becomes

$|y - 1| = 2\sqrt{(x-0)^2 + (y+1)^2 + (z-0)^2} \quad \Rightarrow \quad |y-1| = 2\sqrt{x^2 + (y+1)^2 + z^2} \quad \Rightarrow$

$(y-1)^2 = 4x^2 + 4(y+1)^2 + 4z^2 \quad \Leftrightarrow \quad -3 = 4x^2 + (3y^2 + 10y) + 4z^2 \quad \Leftrightarrow$

$\frac{16}{3} = 4x^2 + 3\left(y + \frac{5}{3}\right)^2 + 4z^2 \quad \Rightarrow \quad \frac{3}{4}x^2 + \frac{9}{16}\left(y + \frac{5}{3}\right)^2 + \frac{3}{4}z^2 = 1$.

This is the equation of an ellipsoid whose center is $\left(0, -\frac{5}{3}, 0\right)$.