22. False. For example, let $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$. Then $|\mathbf{r}(t)| = \sqrt{\cos^2 t + \sin^2 t} = 1 \quad \Rightarrow \quad \dfrac{d}{dt}\,|\mathbf{r}(t)| = 0$, but $|\mathbf{r}'(t)| = |\langle -\sin t, \cos t \rangle| = \sqrt{(-\sin t)^2 + \cos^2 t} = 1$.

23. False. κ is the magnitude of the rate of change of the unit tangent vector $\mathbf{T}$ with respect to arc length s, not with respect to t.

24. False. The binormal vector, by the definition given in Section 10.8, is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = -\,[\mathbf{N}(t) \times \mathbf{T}(t)]$.

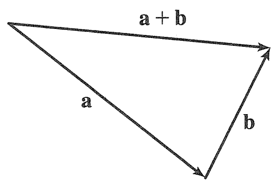
EXERCISES

1. (a) The radius of the sphere is the distance between the points $(-1, 2, 1)$ and $(6, -2, 3)$, namely, $\sqrt{[6-(-1)]^2 + (-2-2)^2 + (3-1)^2} = \sqrt{69}$. By the formula for an equation of a sphere (see page 522), an equation of the sphere with center $(-1, 2, 1)$ and radius $\sqrt{69}$ is $(x+1)^2 + (y-2)^2 + (z-1)^2 = 69$.

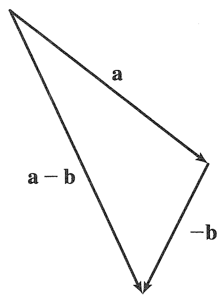
   (b) The intersection of this sphere with the yz-plane is the set of points on the sphere whose x-coordinate is 0. Putting $x = 0$ into the equation, we have $(y-2)^2 + (z-1)^2 = 68$, $x = 0$ which represents a circle in the yz-plane with center $(0, 2, 1)$ and radius $\sqrt{68}$.

   (c) Completing squares gives $(x-4)^2 + (y+1)^2 + (z+3)^2 = -1 + 16 + 1 + 9 = 25$. Thus the sphere is centered at $(4, -1, -3)$ and has radius 5.

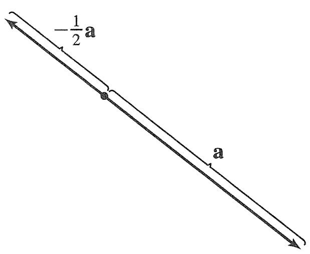
2. (a)



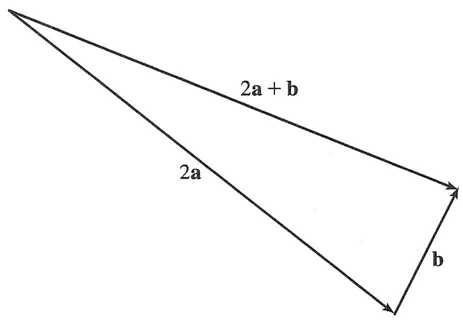
(b)



(c)



(d)



3. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos 45^\circ = (2)(3)\frac{\sqrt{2}}{2} = 3\sqrt{2}$. $\quad |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin 45^\circ = (2)(3)\frac{\sqrt{2}}{2} = 3\sqrt{2}$.
   By the right-hand rule, $\mathbf{u} \times \mathbf{v}$ is directed out of the page.

4. (a) $2\,\mathbf{a} + 3\,\mathbf{b} = 2\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k} + 9\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k} = 11\,\mathbf{i} - 4\,\mathbf{j} - \mathbf{k}$

   (b) $|\mathbf{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

   (c) $\mathbf{a} \cdot \mathbf{b} = (1)(3) + (1)(-2) + (-2)(1) = -1$

(d) $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&-2\\3&-2&1\end{vmatrix}=(1-4)\,\mathbf{i}-(1+6)\,\mathbf{j}+(-2-3)\,\mathbf{k}=-3\,\mathbf{i}-7\mathbf{j}-5\,\mathbf{k}$

(e) $\mathbf{b}\times\mathbf{c}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&-2&1\\0&1&-5\end{vmatrix}=9\,\mathbf{i}+15\,\mathbf{j}+3\,\mathbf{k},\quad |\mathbf{b}\times\mathbf{c}|=3\sqrt{9+25+1}=3\sqrt{35}$

(f) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}1&1&-2\\3&-2&1\\0&1&-5\end{vmatrix}=\begin{vmatrix}-2&1\\1&-5\end{vmatrix}-\begin{vmatrix}3&1\\0&-5\end{vmatrix}-2\begin{vmatrix}3&-2\\0&1\end{vmatrix}=9+15-6=18$

(g) $\mathbf{c}\times\mathbf{c}=\mathbf{0}$ for any $\mathbf{c}$.

(h) From part (e),

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\mathbf{a}\times(9\,\mathbf{i}+15\,\mathbf{j}+3\,\mathbf{k})=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&-2\\9&15&3\end{vmatrix}$$

$$=(3+30)\,\mathbf{i}-(3+18)\,\mathbf{j}+(15-9)\,\mathbf{k}=33\,\mathbf{i}-21\,\mathbf{j}+6\,\mathbf{k}$$

(i) The scalar projection is $\operatorname{comp}_{\mathbf{a}}\mathbf{b}=|\mathbf{b}|\cos\theta=\mathbf{a}\cdot\mathbf{b}/\,|\mathbf{a}|=-\frac{1}{\sqrt{6}}$.

(j) The vector projection is $\operatorname{proj}_{\mathbf{a}}\mathbf{b}=-\dfrac{1}{\sqrt{6}}\left(\dfrac{\mathbf{a}}{|\mathbf{a}|}\right)=-\frac{1}{6}(\mathbf{i}+\mathbf{j}-2\,\mathbf{k})$.

(k) $\cos\theta=\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}=\dfrac{-1}{\sqrt{6}\sqrt{14}}=\dfrac{-1}{2\sqrt{21}}$ and $\theta=\cos^{-1}\left(\dfrac{-1}{2\sqrt{21}}\right)\approx 96^\circ$.

5. For the two vectors to be orthogonal, we need $\langle 3,2,x\rangle\cdot\langle 2x,4,x\rangle=0 \quad\Leftrightarrow\quad (3)(2x)+(2)(4)+(x)(x)=0 \quad\Leftrightarrow$ $x^2+6x+8=0 \quad\Leftrightarrow\quad (x+2)(x+4)=0 \quad\Leftrightarrow\quad x=-2$ or $x=-4$.

6. We know that the cross product of two vectors is orthogonal to both. So we calculate $(\mathbf{j}+2\,\mathbf{k})\times(\mathbf{i}-2\,\mathbf{j}+3\,\mathbf{k})=[3-(-4)]\,\mathbf{i}-(0-2)\,\mathbf{j}+(0-1)\,\mathbf{k}=7\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k}$.

Then two unit vectors orthogonal to both given vectors are $\pm\dfrac{7\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k}}{\sqrt{7^2+2^2+(-1)^2}}=\pm\dfrac{1}{3\sqrt{6}}(7\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k})$,

that is, $\frac{7}{3\sqrt{6}}\,\mathbf{i}+\frac{2}{3\sqrt{6}}\,\mathbf{j}-\frac{1}{3\sqrt{6}}\,\mathbf{k}$ and $-\frac{7}{3\sqrt{6}}\,\mathbf{i}-\frac{2}{3\sqrt{6}}\,\mathbf{j}+\frac{1}{3\sqrt{6}}\,\mathbf{k}$.

7. (a) $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}=\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=2$

(b) $\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v})=\mathbf{u}\cdot[-(\mathbf{v}\times\mathbf{w})]=-\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=-2$

(c) $\mathbf{v}\cdot(\mathbf{u}\times\mathbf{w})=(\mathbf{v}\times\mathbf{u})\cdot\mathbf{w}=-(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}=-2$

(d) $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}=\mathbf{u}\cdot(\mathbf{v}\times\mathbf{v})=\mathbf{u}\cdot\mathbf{0}=0$

8. $(\mathbf{a}\times\mathbf{b})\cdot[(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})]=(\mathbf{a}\times\mathbf{b})\cdot([(\mathbf{b}\times\mathbf{c})\cdot\mathbf{a}]\,\mathbf{c}-[(\mathbf{b}\times\mathbf{c})\cdot\mathbf{c}]\,\mathbf{a})$ [by Theorem 10.4.8, Property 6]

$$=(\mathbf{a}\times\mathbf{b})\cdot[(\mathbf{b}\times\mathbf{c})\cdot\mathbf{a}]\,\mathbf{c}=[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]\,(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$

$$=[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]\,[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]=[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]^2$$

9. For simplicity, consider a unit cube positioned with its back left corner at the origin. Vector representations of the diagonals joining the points $(0,0,0)$ to $(1,1,1)$ and $(1,0,0)$ to $(0,1,1)$ are $\langle 1,1,1\rangle$ and $\langle -1,1,1\rangle$. Let θ be the angle between these two vectors. $\langle 1,1,1\rangle\cdot\langle -1,1,1\rangle = -1+1+1 = 1 = |\langle 1,1,1\rangle|\,|\langle -1,1,1\rangle|\cos\theta = 3\cos\theta \;\Rightarrow\; \cos\theta = \frac{1}{3} \;\Rightarrow$ $\theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 71^\circ$.

10. $\overrightarrow{AB} = \langle 1,3,-1\rangle$, $\overrightarrow{AC} = \langle -2,1,3\rangle$ and $\overrightarrow{AD} = \langle -1,3,1\rangle$. By Equation 10.4.10,

$$\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right) = \begin{vmatrix} 1 & 3 & -1 \\ -2 & 1 & 3 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} - 3\begin{vmatrix} -2 & 3 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} = -8 - 3 + 5 = -6.$$

The volume is $\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right| = 6$ cubic units.

11. $\overrightarrow{AB} = \langle 1,0,-1\rangle$, $\overrightarrow{AC} = \langle 0,4,3\rangle$, so

(a) a vector perpendicular to the plane is $\overrightarrow{AB}\times\overrightarrow{AC} = \langle 0+4, -(3+0), 4-0\rangle = \langle 4,-3,4\rangle$.

(b) $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\sqrt{16+9+16} = \frac{\sqrt{41}}{2}$.

12. $\mathbf{D} = 4\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k}$, $\quad W = \mathbf{F}\cdot\mathbf{D} = 12 + 15 + 60 = 87$ J

13. Let F_1 be the magnitude of the force directed 20° away from the direction of shore, and let F_2 be the magnitude of the other force. Separating these forces into components parallel to the direction of the resultant force and perpendicular to it gives $F_1\cos 20^\circ + F_2\cos 30^\circ = 255$ **(1)**, and $F_1\sin 20^\circ - F_2\sin 30^\circ = 0 \;\Rightarrow\; F_1 = F_2\dfrac{\sin 30^\circ}{\sin 20^\circ}$ **(2)**. Substituting **(2)** into **(1)** gives $F_2(\sin 30^\circ\cot 20^\circ + \cos 30^\circ) = 255 \;\Rightarrow\; F_2 \approx 114$ N. Substituting this into **(2)** gives $F_1 \approx 166$ N.

14. $|\tau| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = (0.40)(50)\sin(90^\circ - 30^\circ) \approx 17.3\ \mathrm{N\cdot m}$.