A Refinement Proof of $\boxed{6.40}$

For $n=0$, set $S_0 = \phi \subset S$ s.t. $\sum_{i \in S_0} i = 0$.

Assume for $n=k$ $\exists S_n \subset S$ s.t. $\sum_{i \in S_k} i = k$

Then for $n=k+1$, find $a = \min(S - S_k)$, since $a \in S$, $a = 2^p$ for some $p \geq 0, p \in \mathbb{Z}$.

If $p=0$, then $S_k \cup \{2^0\} = S_{k+1}$, where $\sum_{i \in S_{k+1}} i = \sum_{j \in S_k} j + 1 = k+1$

If $p \geq 1$, then set $S_{k+1} = \left(S_k - \{2^j \mid 0 \leq j \leq p-1\}\right) \cup \{2^p\}$ s.t.

$$\sum_{i \in S_{k+1}} i = \sum_{j \in S_k} j - (2^0 + 2^1 + \cdots 2^{p-1}) + 2^p = k - (2^p - 1) + 2^p = k+1$$

Therefore, by mathematical induction, the statement is true for every $n \in \mathbb{N}$

Note: $A = 2^0 + 2^1 + \cdots + 2^{p-1}$, then $2A - A = 2^p - 2^0 = 2^p - 1$

$2A = \quad 2^1 + \cdots + 2^{p-1} + 2^p$

A Refinement Proof of $\boxed{6.57}$

(a) $P(1)$: Let $S_1 = \{1\} \subset S$, then $\sum_{i \in S_1} i = 1$ (True)

(b) If $1 \leq k < 300$, $\exists S_k \subset S$ s.t. $\sum_{i \in S_k} i = k$, then find $a = \min(S - S_k)$.

$\boxed{\text{Case 1}}$ If $a = 1$, then set $S_{k+1} = S_k \cup \{1\}$ s.t. $\sum_{j \in S_{k+1}} j = \sum_{i \in S_k} i + 1 = k+1$

$\boxed{\text{Case 2}}$ If $2 \leq a \leq 24$, then set $S_{k+1} = \left(S_k - \{a-1\}\right) \cup \{a\}$ s.t.

$$\sum_{j \in S_{k+1}} j = \sum_{i \in S_k} i - (a-1) + a = \sum_{i \in S_k} i + 1 = k+1$$

Therefore, by ~~finite it induction of mathematical~~

finite mathematical induction, the statement is true.

(6.34) Assume exist a number $n \in \mathbb{N}$. s.t. $3 \nmid 2^{2n}-1$.

Then exist a mimnum number $m \in \mathbb{N}$ s.t. $3 \nmid 2^{2m}-1$

~~Since $3 \nmid 2$~~ Then for $k = m-1$ $3 \mid 2^{2k}-1$, assume $2^{2k-1} = 3q$

Then $2^{2(k+1)}-1 = [2^2 \cdot 2^{2k} - 2^{2k}] + [2^{2k}-1] = 2^{2k}(3) + 3q = 3(2^{2k}+q)$

A contradiction to ~~minimum~~ assumption, n.

Thus, $3 \mid 2^{2n}-1$ $\forall n>0$, $n \in \mathbb{N}$. #

(6.40) Assume $\underbrace{S_n \subseteq S}_{\exists n \in \mathbb{N}}$ s.t. $S_n \subseteq S$ and $\sum_{i \in S_n} \neq n$

Then $\exists m \in \mathbb{N}$, m is minimal number s.t. $S_m \subseteq S$ and $\sum_{i \in S_n} \neq n$

Then $\exists S_{m-1} \subseteq S$ s.t. $\sum_{i \in S_n} i = m+1$

Then there is a minimal number q, $0 \leq q \leq m$ s.t.

$2^{q+1} \notin S_{m-1}$ and $(2^0, 2^1, 2^2, \ldots, 2^q \in S_{m-1})$, where $2^0+2^1+\cdots+2^q = 2^{q+1}-1$

Set $S_m = (S_{m-1} - \{2^0, 2^1, \cdots 2^q\}) \cup \{2^{q+1}\}$

Then $\sum_{i \in S_m} i = \left(\sum_{j \in S_{m-1}} j\right) - (2^0+2^1+\cdots+2^q) + 2^{q+1}$

$= \left(\sum_{j \in S_{m-1}} j\right) - (2^{q+1}-1) + 2^{q+1} = \sum_{j \in S_{m-1}} j + 1 = m$

Contradict with assumption n #.

(6.41) Conjecture(推测) $a_n = 2^{n-1}$ for all $n \geq 1$

~~Proof:~~ $a_1 = 2^{1-1} = 2^0 = 1$ (True)

Assume $a_k = 2^k$, then $a_{k+1} = 2 \cdot 2a_k = 2 \cdot 2^{k-1} = 2^k$.

By mathematic induction, it's true for $k \geq 1$.

(6.45) Question: $\forall n \geq 12.\ n \in \mathbb{N}.\ \exists a \geq 0\ b \geq 0\ a, b \in \mathbb{N}$ s.t. $n = 3a + 7b$

If $n = 12$, $12 = 3 \cdot 4 + 7 \cdot 0$

Assume ~~$\exists k \in \mathbb{N}$ s.t~~ $\exists k \in \mathbb{N}$ s.t, $\forall i \in \mathbb{N}$, $12 \leq i \leq k$

There exist nonnegative integers a and b s.t. $i = 3a + 7b$

Since $12 \leq k-2 \leq k$, $(13 = 3 \cdot 2 + 7,\ 14 = 3 \cdot 0 + 7 \cdot 2)$

$\exists c, d \in \mathbb{N}$, $c, d \geq 0$ s.t, $k-2 = 3c + 7d$

then $k+1 = 3(c+1) + 7d$.

By strong ~~p~~ mathematical Induction, the assumption is true.

(6.57) $P(1)$: $1 \leq 1 \leq 300$, exists a subset $S_1 = \{1\} \subseteq S$ s.t $\sum_{i \in S_1} i = 1$ (True)

If $P(k)$, (i.e. $1 \leq k \leq 300$, exists a subset $S_k \subseteq S$ s.t $\sum_{i = S_k} i = k$)

then we need to show that $1 \leq k+1 \leq 300$, exists a subset $S_{k+1} \subseteq S$

s.t. $\sum_{p = S_{k+1}} p = k+1$. Assume there exist a smallest integer $1 \leq m \leq 24$ s.t. $\{24, 23 \cdots m\} \subseteq S_k$ and $m-1 \notin S_k$ and a largest integer $0 \leq r \leq m$ and ($r = 0$ or $r \in S_k$)

Then set $S_{k+1} = (S_k - \{r\}) \cup \{r+1\}$

Then $\sum_{p \in S_{k+1}} p = \sum_{i \in S_k} i - r + r + 1 = k+1$ for $1 \leq k \leq 300$.

By finite mathematical induction, the statement is true.

6.61

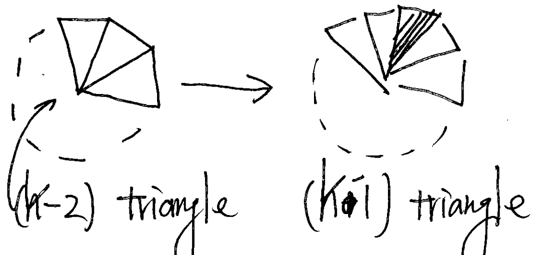
$n=3$, a triangle, sum of the interior ~~agles~~ angles of it is $(3-2)\cdot 180^\circ = 180^\circ$ (True)

Assume ~~k-gon~~, the sum of the interior angles of k-gon is

$\boxed{180^\circ (k-2)}$

Then for (k+1)-gon, as figure shows.

k-gon



there are $(k-2)+1$ triangle in (k+1)-gon.

then the sum of the interior angles of (k+1)-gon is

$(k-1)\cdot 180^\circ$.

By mathematical induction, the ~~assumption~~ statement is true.