3,31

~~a+b~~

Case1 $a+b$ and ab are even

then a and b are even.

Case2 $a+b$ and ab are odd.

then a and b are odd by ab odd.

~~the~~ but $a+b$ will be even. $\divideontimes$

3,35.

Case1. If $n=0$, then $2^0+6^0=2$. (even)

Case2 If $n\geq 1$, then $2^n+6^n=2(2^{n-1}+3\cdot 6^{n-1})$ (even)

Homework 5

4.4 Then the possible remainder of x and y divided by 3 are,

x	y	x^2	y^2	x^2-y^2
1	1	1	1	0
1	2	1	$4\equiv 1$	0
2	1	$4\equiv 1$	1	0
2	2	$4\equiv 1$	$4\equiv 1$	0

, That is, $3 \mid x^2-y^2$ #

4.10

$4 \mid n^2+3$, then $\exists m \in \mathbb{Z}$ s.t. $4m = n^2+3$

$\Leftrightarrow 4m(n^2-3) = (n^2+3)(n^2-3)$

$\Leftrightarrow 4m(n^2-3)+6 = (n^4-9)+6 = n^4-3$

$\Leftrightarrow 2(m(n^2-3)+3) = n^4-3$

$\Leftrightarrow 2 \mid n^4-3$

4.13 Lemma 1. If $3 \mid ab$, then $3 \mid a$ or $3 \mid b$.

Lemma 2, If $3 \nmid (x^2-1)$, then $3 \mid x$

Lemma 3, If $c \in \mathbb{Z}$, then $c^2 \equiv 0 \pmod 3$ or $c^2 \equiv 1 \pmod 3$

Pf:/ Since $4\times 4 = 16$, $16-1=15$, $3 \mid 15$, So $\{c \in \mathbb{Z} \mid 3 \mid c^2-1\} \neq \emptyset$.

But If $3 \nmid c^2-1$, then $3 \mid c$ by lemma 2, then $\exists m \in \mathbb{Z}$ s.t. $3m = c$

$\Rightarrow 3\cdot 3m^2 = x^2$, then $3 \mid c^2$. That is, neither $c^2 \equiv 0 \pmod 3$ or $c^2 \equiv 1 \pmod 3$

pf:/ Assume $3 \nmid ab$, then $3 \nmid a$ and $3 \nmid b$, then $3 \nmid a^2$ and $3 \nmid b^2$.

By Lemma 3, $3 \mid a^2-1$ and $3 \mid b^2-1 \Rightarrow 3 \mid a^2+b^2-2 \Rightarrow 3 \mid c^2-2$

That is $c \equiv 2 \pmod 3$, which contradict with lemma 3.

That means, $a^2+b^2 \neq c^2$ #

~~$16k^2-8k+1 \equiv 3$~~

~~$16k^2-8k-2$~~

4.16

	a	b	a^2+2b^2
✓	0	0	0
	0	1	2
	0	2	2
	1	0	1
	1	1	0
✓	1	1	0
✓	1	2	0
	2	0	1
✓	2	1	0
✓	2	2	0

That is, if $a^2+2b^2 \equiv 0 \pmod 3$ Then a and b are both congruent to 0 module 3 or neither is congument to 0 module 3.

4.24

x mod4	y mod4	$x^2 \equiv y^2 \pmod{16}$
0	0	True
2	0	False
0	2	False
2	2	True

4.30 $|xy| = -xy$, then $|x| = -x$, $|y| = y$

or $|x| = x$, $|y| = -y$

Then $|xy| = -xy = |x||y| = (-x)(y)$ or $(x)(-y)$

$|xy| = xy$, then $|x| = x$ $|y| = y$

or $|x| = -x$ $|y| = -y$

Then $|xy| = xy = |x||y| = (-x)(-y) = xy$

4.31

$|x| = |(x+y)+(-y)| \leq |x+y| + |-y| = |x+y| + |y|$

$\Rightarrow |x+y| \geq |x| - |y|$

4.33 Assume $a = r^3$ $b = s^3$ $c = t^3$

Then $r^3+s^3+t^3-3rst = \frac{1}{2}\underbrace{(r+s+t)}_{(+)}\left[\underbrace{(r-s)^2+(s-t)^2+(t-r)^2}_{(+)}\right] \geq 0$

Then $r^3+s^3+t^3 \geq 3rst$

$\Leftrightarrow \frac{a+b+c}{3} \geq \sqrt[3]{abc}$ //

4.45. $\forall k \in B$, $k = 4m+3 = 2(2m+1)+1$

Then $k \equiv 1 \pmod 2$, then $k \in A$.

Thus, $B \subseteq A$.

4.48 ~~$\forall x \in A$, $x = 2n$,~~ ~~$\forall y \in B$, $x = 4k$~~

($\Rightarrow$) $n \in A - B$, then $n = 2k$ and $2 \nmid k$

Otherwise $n = 2(2m)$, where $k = 2m$

then $n = 4m \in B$ $*$

Thus, k is odd.

($\Leftarrow$) k odd, then $n = 2k$ $2 \nmid k$, thus $4 \nmid n$, $n \notin B$

$n \in A$.

4.57 By theorem 4.22:

$$\overline{\overline{A}\cup(\overline{B}\cap C)} = \overline{\overline{A}}\cap(\overline{\overline{B}\cap C}) = A\cap(\overline{\overline{B}}\cup\overline{C})$$

$$= A\cap(B\cup\overline{C}) = (A\cap B)\cup(A\cap\overline{C})$$

$$= (A\cap B)\cup(A-C)$$

4.61 For $A=\{1\}$, $B=\{2\}$, $P(A)=\{\emptyset, A\}$. and $P(B)=\{\emptyset, B\}$. Thus,

$P(A)\times P(B) = \{(\emptyset,\emptyset),(\emptyset,B),(A,\emptyset),(A,B)\}$

Since $A\times B = \{(1,2)\}$, $P(A\times B) = \{\emptyset, A\times B\}$

4.68

Let $(x,y) \in (A\times B)\cap(C\times D)$

Then $x \in (A\cap C)$ $y \in (B\cap D)$ $\Leftrightarrow$ $x \in (A\cap C)$, $y \in (B\cap C)$ //

$\Leftrightarrow$ $(x,y) \in (A\cap C)\times(B\cap D)$

#70 A Let $U = \{1, 2, 3, 4\}$

$A = \{1, 2\}$, $B = \{2, 3, 4\}$

$\overline{A \times B} = \{ (1,1), (2,1), (3,1), (3,2), (3,3), (3,4)$
$(4,1)\ (4,2)\ (4,3)\ (4,4) \}$

$\bar{A} \times \bar{B}\ \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\quad (2,1)\ (2,2)\ (2,3)\ (2,4)$
$(3,2)\ (3,3)\ (3,4)\quad (4,2)\ (4,3)\ (4,4) \}$

$\overline{A \times B} \neq \bar{A} \times \bar{B}$