Homework 4

3.4

~~$x^2 - 5x - 1$~~ ~~Since~~ $(x-1)(x-3) = -2 \Rightarrow x^2 - 4x + 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16-20}}{2}$$, which do not exist.

Take $x = 1$, $1^2 - 4 + 5 = 2 > 0$

That means, $x^2 - 4x + 5 > 0 \quad \forall x \in \mathbb{R}$

$\Leftrightarrow (x-1)(x-3) \geq -2 \ \forall x \in \mathbb{R}$.

~~Thus~~, Since $\{x \mid x^3 - 5x - 1 \geq 0\} \subset \mathbb{R}$, if $x^3 - 5x - 1 \geq 0$,

then $(x-1)(x-3) \geq -2$ #

3.7

$$(x-y)^2 + (y-z)^2 + (z-x)^2 = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz \geq 0$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 \geq 2xy + 2yz + 2zx \Rightarrow x^2 + y^2 + z^2 \geq xy + xz + yz$$

Thus, $x^2 + y^2 + z^2 < xy + xz + yz$ is false statement $\forall x, y, z \in \mathbb{R}$.

That proof $x^2 + y^2 + z^2 < xy + xz + yz$, then $x + y + z > 0$.

3.11

If $1 - n^2 > 0$, then $1 > n^2$, then $n = 0$.

Thus $3n - 2 = 3 \cdot 0 - 2 = -2$, -2 is an even integer.

3.14

	$\frac{n^2+n-6}{2}$ is odd	$\frac{2n^3+3n^2+n}{6}$ is even	Statement
1	-2 (F)		T
5	13 (T)	55 (F)	F
9	42 (F)		T

Ans: the statement is true for $n = 1, 9$ #

3.18

($\Leftarrow$) If x is odd, $5x$ is odd, then $5x+(-11)$ is even.
(odd + odd)

($\Rightarrow$) If $5x+(-11)$ is even, then $\underset{\text{even}}{(5x-11)}+\underset{\text{odd}}{11}$ is odd.

Then x is odd since $5x$ is odd. #

3.24

($\Leftarrow$) If $\cos\frac{n\pi}{2}$ is even, then $\cos\frac{n\pi}{2}=0$, then $n=\overset{-1}{1}, \overset{-3}{3}, \overset{-5}{5}, \overset{-7}{7}, \overset{\cdots}{\cdots}$; n is odd

$\Rightarrow$ $\underset{\text{(even)(odd)}}{2\cdot n^2}+\underset{\text{(odd)}}{n}$ is odd. #

($\Rightarrow$) Proof by Contrapositive $\cos\frac{n\pi}{2}$ is odd, then n is even.

$\Rightarrow$ $2\cdot n^2+n$ is even #

3.51

	$(x+1)y^2$
x is odd, y is even	even
x is even, y is even	even
x is odd, y is odd	even

($\Leftarrow$)

($\Rightarrow$) Proof by Contrapositive

x is even and y is odd, then $\underset{\text{odd}}{(x+1)}\cdot\underset{\text{odd}}{y^2}$ is odd #

3.56

	$ax+by$
x even y even	even
x odd y odd	even
x even y odd	even
x odd y even	even

#

3.65 Let $a=2n+1$, then $[(2n+1)^2+3][(2n+1)^2+7]=32b$

$\Rightarrow 4(n^2+n+1)\cdot 4(n^2+n+2)=32b \Rightarrow \underset{\text{odd}}{(n^2+n+1)}\underset{\text{even}}{(n^2+n+2)}=2b$

Then $b=n^2+n+1\cdot\left(\frac{n^2+n+2}{2}\right)$, where $a=2n+1$. #