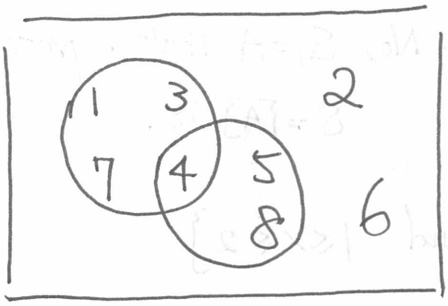
Homowork 2.

1.13



1.24

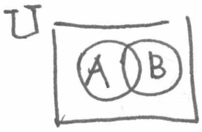
$B = \{1.2.3.4\}$

$C = \{2,3,4,5\}$

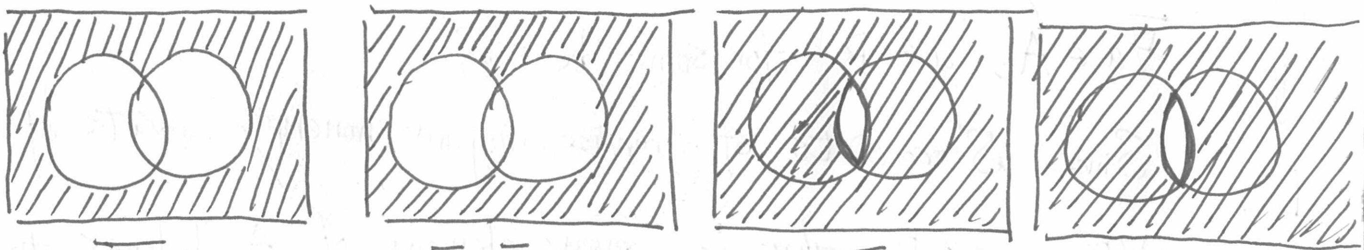
$A = \{1 \quad 5,6\}$

$B - A = \{2,3,4\} = C - A$

1.26.



(a)



$\overline{A \cup B}$ $\qquad$ $\overline{A} \cap \overline{B}$ $\qquad$ $\overline{A \cap B}$ $\qquad$ $\overline{A} \cup \overline{B}$

1.29.

(a) $\phi. \{\phi\} \in A$, $\{\phi. \{\phi\}\} \notin A$. $\qquad$ (b) $|A| = 3$

(c) $\phi, \{\phi\}, \{\phi, \{\phi\}\}$ are subsets of A

(d) $\phi \cap A = \phi$ (e) $\{\phi\} \cap A = \{\phi\}$, (f) $\{\phi, \{\phi\}\} \cap A = \{\phi, \{\phi\}\}$.

(g) $\phi \cup A = A$ (h) $\{\phi\} \cup A = A$, (i) $\{\phi, \{\phi\}\} \cup A = A$.

1.32 $A = \{1,2,3,4\}$, $B = \{1,2,4\}$ $C = \{1,3\}$ $D = \{2,3\}$

Then $A \cap B = \{1,2,4\}$ $A \cap C = \{1,3\}$ $A \cap D = \{2,3\}$

$B \cap C = \{1\}$ $B \cap D = \{2\}$ $C \cap D = \{3\}$ #

~~Note: The key to find A, B, C, D is that~~

~~make sure $A \cup B \cup C \cup D$~~

Further question: How to find all combination of these sets?

How ~~may~~ many combinations ~~the~~ are they?

1.47

(a) No, since there is no 4.

(b) Yes

(c) No, since $\{1,2\} \cap \{2,3\} = \{2\}$

(d) ~~Yes.~~ No, $S_4 = A$ isn't a partition of A
$S = \{A\}$ is.

1.51

$\{x \mid x \in Q \text{ and } x < 1\}$, $\{x \mid x \in Q \text{ and } 1 \le x \le 2\}$
$\{x \mid x \in Q \text{ and } x > 2\}$.

1.56

Since every element of A belong to a subset in S,

D1 $\Downarrow$ D2

$\forall a \in A$, $a \in B$ for some $B \in S$

Since S consists of pairwise disjoint nonempty subsets of A

$\forall B'$, $a \notin B'$, that is, every element of A belongs to exactly one subset in S

D2 $\Downarrow$ D3

The collection S consists of nonempty subsets of A

$\Rightarrow$ (1) every subset in S is nonempty.

Every element of A belong to exactly one subset in S

$\forall B \in S$ and $\exists b \in B$, then pick any $B' \in S$

If $b \in B \cap B'$ then $\forall b' \in B$, $b' \in B' \Rightarrow B \subset B'$
and $\forall b' \in B'$, $b' \in B \Rightarrow B' \subset B$ $\Rightarrow B = B'$

Otherwise, it will contradict to "every element of A belong to exactly one subset in S

Therefore, every two subsets of A are equal or disjoint. (2)

Since every element of A belong to exactly one subset in S.

The union of all subsets in S is A (3)

(to be continued)

(continued) Condition (1) plus (2), we have S consists of pairwise disjoint nonempty subsets of A.

D3

$\Downarrow$

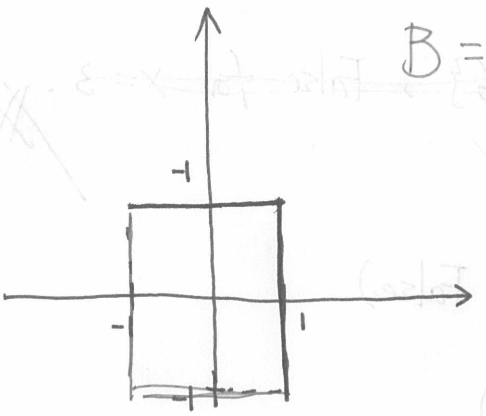
D1

Condition (3), we have every element of A belongs to a subset in S.

1.64 $A \times B = \{(1,1), (1,2)\}$, then $\mathcal{P}(A \times B) =$

$\{\{(1,1),(1,2)\}, \{(1,1)\}, \{(1,2)\}, \emptyset\}$

1.66



$B = \{-1, +1\}$ $A = \{x \mid -1 \le x \le 1\}$

~~It is a line bends in 90 degrees~~.

It is a hollow square.

1.85 $S = \{x \in \mathbb{R} \mid x^2 + 2x - 1 = 0\} = \{-1+\sqrt{2}, -1-\sqrt{2}\}$

$$B = A_{(-1-\sqrt{2})} \times A_{(-1+\sqrt{2})} = \left\{ \begin{matrix} (-1-\sqrt{2}, \sqrt{2}) & (-\sqrt{2}, \sqrt{2}) \\ (-1-\sqrt{2}, -1+\sqrt{2}) & (-\sqrt{2}, -1+\sqrt{2}) \end{matrix} \right\}$$

Let $c = \sqrt{2}$, then

$C = \{-a-a^2, -a^2, 1-a^2, a-a^2\}$

Then $\cancel{-a}-a^2-a^2+1-a^2\cancel{+a}-a^2 = -4a^2+1 = -7$ #

Chapter 2

(a) ~~$P(x) \Rightarrow Q(x)$ is true for $x = -4, 4$; False for $x = -3, 1, 5$~~

~~(b)~~

Chapter 2

2,31

(a) $P(x) \Rightarrow Q(x)$ is true for $x = -3, 1, 4, 5$; False for $x = -4$

(b) $P(x) \Rightarrow Q(x)$ is true for $x \in S$

(c) ~~$P(x) \Rightarrow Q(x)$ is true for $x \in \{0, 2, 4, 6\}$; False for $x = 3$.~~

$P(x) \Rightarrow Q(x)$ is true for $x \in S$

2,35

P: 18 is odd. (False) Q: 25 is even (False)

Then, $P \Rightarrow Q$ (~~False~~ True) and $Q \Rightarrow P$ (True)

Then $P \Leftrightarrow Q$ is true.

2,62

P	Q	$P \vee Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge (\sim P \wedge Q)$ ☆1	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$	$\sim(P \Leftrightarrow Q)$ ☆2
T	T	T	F	F	T	T	T	F
T	F	T	T	T	F	T	F	T
F	T	T	T	T	T	F	F	T
F	F	F	T	F	T	T	T	F

☆1 $\equiv$ ☆2

2.68.

(a) Exists a rational number r, the number $1/r$ is not irrational.

(b) For every rational number r, $r^2 \neq 2$

2.71

~~$\exists a, \exists b$ s.t. $(ab<0) \wedge (a+b$~~

(a) $\exists a \in \mathbb{Z}\ \exists b \in \mathbb{Z}$ s.t. $(ab>0) \wedge (a+b>0)$.

(b) $\forall x, y \in \mathbb{R}\ (x \neq y) \Rightarrow x^2+y^2>0$

~~(c)~~

(d) $\forall a \in \mathbb{Z}\ \forall b \in \mathbb{Z}$ s.t. $(ab \leq 0) \vee (a+b \leq 0)$

$\exists x, y \in \mathbb{R}$ s.t. $(x \neq y) \wedge (x^2+y^2 \leq 0)$

(c) For all integer a and b either $ab \geq 0$ or $a+b>0$.

There exists a real number x and y s.t. $x \neq y$ and $x^2+y^2 \leq 0$.

$\#$