

1.2

$$\text{Let } S = \{-2, -1, 0, 1, 2, 3\}$$

$$(a) A = \{1, 2, 3\} = \{x \in S : x > 0\}$$

$$(b) B = \{0, 1, 2, 3\} = \{x \in S : x \geq 0\}$$

$$(c) C = \{-2, -1\} = \{x \in S : x < 0\}$$

$$(d) D = \{-2, 2, 3\} = \{x \in S : |x| > 1\}$$

1.5

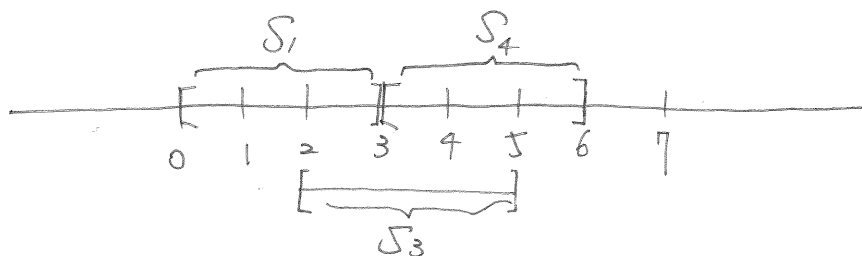
$$(a) A = \{-1, -2, -3, \dots\} = \{x \in \mathbb{Z} : x \leq -1\}$$

$$(b) B = \{-3, -2, \dots, 3\} = \{x \in \mathbb{Z} : |x| \leq 3\}$$

$$(c) C = \{-2, -1, 1, 2\} = \{x \in \mathbb{Z} : 0 < |x| < 3\}$$

1.36

$$S_1 = [0, 3] \quad S_3 = [2, 5] \quad S_4 = [3, 6]$$



$$\bigcup_{\alpha \in A} S_\alpha = S_1 \cup S_3 \cup S_4 = [0, 6]$$

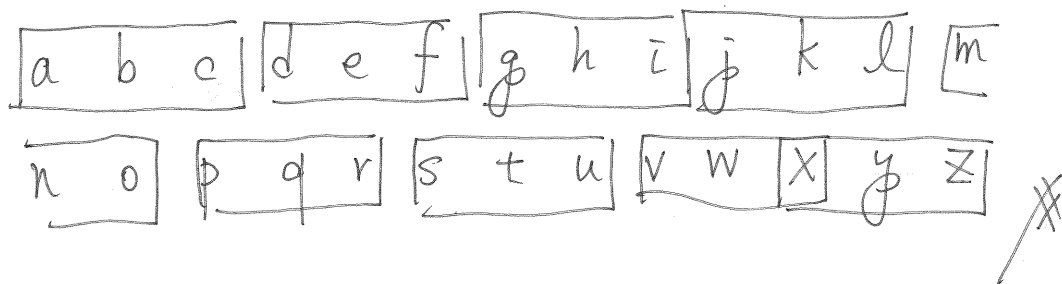
$$\bigcap_{\alpha \in A} S_\alpha = S_1 \cap S_3 \cap S_4 = \{3\}$$

1.39

To make  $S$  has smaller cardinality, we have to make  $A_\alpha \cap A_\beta = \emptyset \quad \forall \alpha, \beta$  as possible as we can. Unfortunately,

In words, there are 26 letters, and each  $A_\alpha, \forall \alpha \in S$  containing 3 letters  $26 \div 3 = 8 \dots 2$ . Therefore, although we can't make  $A_\alpha \cap A_\beta = \emptyset \quad \forall \alpha, \beta \in S$ ,  $|S| = 9$  is the best, smallest cardinality number we could achieve.

In figure,



1.42

(a)

Since  $A_1 \supset A_2 \supset A_3 \dots$ ,  $\bigcup_{n \in \mathbb{N}} A_n = [1, 3)$

$\forall x > 2 \exists n$  large enough s.t.  $(2 + \frac{1}{n}) < x$ ,

therefore,  $\bigcap_{n \in \mathbb{N}} [1, 2 + \frac{1}{n}) = [1, 2]$

(b)

Since  $A_1 \subset A_2 \subset A_3 \dots$ ,  $\bigcap_{n \in \mathbb{N}} A_n = (-1, 2)$

$A_n = (\frac{-2n-1}{n}, 2n)$ , as  $n \rightarrow \infty$ ,  $2n \rightarrow \infty$

and  $-\frac{2n-1}{n} \rightarrow -2$

Thus,  $\bigcup_{n \in \mathbb{N}} A_n = (-2, \infty)$

1,78

(a)  $\bigcup_{r \in I} A_r = \{(x, y) \in \mathbb{R} \times \mathbb{R}\}$ ,  $\bigcap_{r \in I} A_r = \emptyset$ .

(b)  $\bigcup_{r \in I} B_r = \{(x, y) \in \mathbb{R} \times \mathbb{R}\}$ ,  $\bigcap_{r \in I} B_r = \{(0, 0)\}$

(c)  $\bigcup_{r \in I} C_r = \{(x, y) \in \mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}\}$ ,  $\bigcap_{r \in I} C_r = \emptyset$ .

---

2,2

(a) (True) (b) D is Fibonacci number,  $33 \notin D$  (False)

(c)  $22 = 1 + 3 \cdot 7 \in A$  (False)

(d) (True) (e)  $3 \in B \cap D$  (False)

(f) 53 is prime number (False).

2,4

$$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) \Rightarrow x = 3, -2$$

(a) 3, -2 is true for  $P(x)$ .

(b)  $\mathbb{R} \setminus \{3, -2\}$  is false for  $P(x)$ .

2,7

Here are first six twin primes

(3, 5) (5, 7), (11, 13), (17, 19), (29, 31), (41, 43)

2,14

(a) The number of my overdue library books is less than 2.

(b) (Both my two friends misplaced his homework assignment.) or  
(None of my two friends misplaced his homework assignment.)

(c) Someone expected that to happen.

(d) My instructor often teaches that course.

(e) It's not surprising that two students received the same exam score.

$\boxed{2,17}$   $\phi$  (True)  $Q$ : (False)

(a)  $(T \vee F) = T$  (b)  $(T \wedge F) = F$  (c)  $(\sim T) \vee F = F$

(d)  $T \wedge (\sim F) = T$

---

$\boxed{1,12}$   $A = B = C = D = E$

$\boxed{1,15}$   $\mathcal{P}(A) = \{\phi, \{0\}, \{\{0\}\}, A\}$

$\boxed{1,20}$  (a) False, counterexample  $A = \{1, \{1\}\}$

(b) True. (c) False, counterexample  $B = \{1\}$ ,  $A = \phi$   $|P(B)| = 2$   $|P(A)| = 1$

(d) True.

$\boxed{1,28}$  (a)  $A = \{1\}$ ,  $B = \{\{1\}\}$ ,  $C = \{1, 2\}$ .

(b)  $A = \{\{1\}, 1\}$ ,  $B = \{1\}$ ,  $C = \{1, 2\}$  (c)  $A = \{1\}$ ,  $B = \{\{1\}\}$ ,  $C = \{\{1\}, 2\}$

$\boxed{1,36}$   $\boxed{1,39}$   $\boxed{1,42}$

$\boxed{1,43}$   $\bigcup_{r \in \mathbb{R}^+} A_r = \mathbb{R}$ ,  $\bigcap_{r \in \mathbb{R}^+} A_r = \{0\}$   $\boxed{1,45}$   $\bigcup_{n \in \mathbb{N}} A_n = (-1, 2)$   $\bigcap_{n \in \mathbb{N}} A_n = [0, 1]$

$\boxed{1,78}$

---

$\boxed{2,2}$   $\boxed{2,4}$   $\boxed{2,7}$   $\boxed{2,14}$

$\boxed{2,16}$   $P$ : False,  $Q$ : True (a) T (b) F (c) F (d) T (e) T

$\boxed{2,20}$   $\boxed{2,25}$

2, 49

P	Q	R	A $\equiv$ P $\Rightarrow$ Q	B $\equiv$ Q $\Rightarrow$ R	C $\equiv$ P $\Rightarrow$ R	D $\equiv$ A $\wedge$ B	D $\Rightarrow$ C
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

There is, the statement is a tautology.

2, 91

P	Q	R	A $\equiv$ P $\wedge$ Q	$\alpha$ $\equiv$ A $\Rightarrow$ R	B $\equiv$ P $\wedge$ $\neg$ R	$\beta$ $\equiv$ B $\Rightarrow$ $\neg$ Q	C $\equiv$ Q $\wedge$ (R)	$\gamma$ $\equiv$ C $\Rightarrow$ $\neg$ P
T	T	T	T	T	F	T	F	T
T	T	F	T	F	T	F	T	F
T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	T	F	T
F	T	T	F	T	F	T	F	T
F	T	F	F	T	F	T	T	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

$\alpha \equiv \beta \equiv \gamma$

