Homework 12

10.20

We know $\mathbb{R} = \mathbb{Q} \cup (\mathbb{R}\setminus\mathbb{Q})$, where $\mathbb{Q}$ is denumberable. $(\mathbb{R}\setminus\mathbb{Q})$ is the set of irrational numbers.

If $(\mathbb{R}\setminus\mathbb{Q})$ is denumberable, ~~the~~ $\mathbb{R} = \mathbb{Q} \cup (\mathbb{R}\setminus\mathbb{Q})$ is denumberable. Which contradicts with the fact that $\mathbb{R}$ is uncountable. Thus $(\mathbb{R}\setminus\mathbb{Q})$ is uncountable.

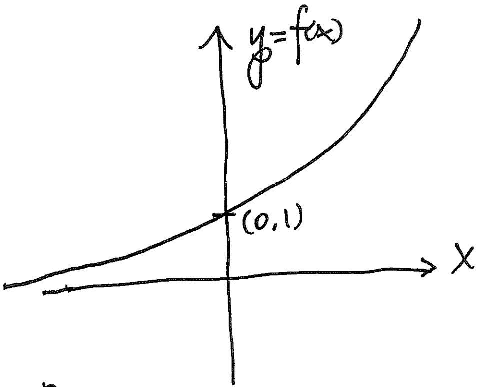
10.21

Complex numbers contains $\mathbb{R}$, as $\mathbb{R}$ is uncountable. $\mathbb{C}$ is uncounatble.

10.24

Let $f: \mathbb{R} \to \mathbb{R}^+$ defined in $f(x) = 2^x$

In graphic, it is clearly that f is bijection.



Therefore, $\mathbb{R}$ and $\mathbb{R}^+$ are numerically equivalent.

10.27 Let $b \in B$, Then $f: A \to A \times B$ defined by $f(a) = (a,b)$ $\forall a \in A$. It's clearly 1-1, Thus, $|A| \leq |A \times B|$

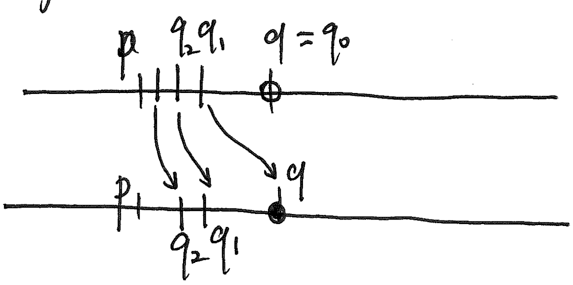
10.32

By Schröder-Bernstein Therem. Since $|A| \leq |B| \leq |C| = |A|$.

$|A| = |B|$

10.34

Let $f: (\mathbb{Q} - \{q\}) \to \mathbb{Q}$ defined by following figure.



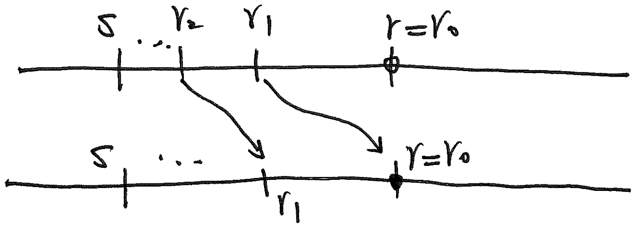
$p \in \mathbb{Q}$ and $p < q = q_0$

$q_i = \frac{q_{i-1} + p}{2}$ for $i \geq 1$ $i \in \mathbb{N}$

$$f(x) = \begin{cases} q_i & \text{if } x = q_{i-1} \\ x & \text{otherwise.} \end{cases}$$

It's easy to check that f is bijection. Thus $|\mathbb{Q} - \{q\}| = |\mathbb{Q}| = \aleph_0$

Same technique. let $g: (\mathbb{R} - \{r\}) \to \mathbb{R}$ defined by following figure.



$s \in \mathbb{R}$ and $s < r = r_0$

$r_i = \frac{r_{i-1} + s}{2}$ for $i \geq 1$ $i \in \mathbb{N}$.

$$g(x) = \begin{cases} r_i & \text{if } x = r_{i-1} \\ x & \text{otherwise.} \end{cases}$$

g is bijection. Thus $|\mathbb{R} - \{r\}| = |\mathbb{R}| = c$.

<u>10.42</u> $|S-T| = |T-S|$, then $\exists f, g$ s.t. f, g are bijection.

$f : (S-T) \to (T-S)$.

$g : (T-S) \to (S-T)$.

Then set $h : S \to T$ by $h(s) = \begin{cases} s. & \text{if } s \in T \cap S \\ f(s) & \text{if } s \in S-T \end{cases}$

h is bijection since $f(s)$, $i_{T \cap S}$ are bijection.

Then $|S| = |T|$

<u>10.49</u>

(a) $\bigcup_{n \in \mathbb{Z}} \underline{(n, n+1]}$ uncountable

(b) $\bigcup_{r \in \mathbb{R}'} \{r+q \mid q \in \mathbb{Q}\}$

where $\mathbb{R}' = \mathbb{R} \setminus (\mathbb{Q} - \{0\})$