國立成功大學八十一學年度轉學生考試(微積分試題)	共 1 頁 第 1 頁

1. (i) Suppose $u = f(x,y,z)$, $z = g(x,y,t)$ and $y = h(x,t)$ are differentiable real-valued functions in suitable domains. Find $\frac{\partial u}{\partial x}(x,t) =?$ $\frac{\partial u}{\partial t}(x,t) =?$ 10%

   (ii) Suppose $f : \mathbf{R}^2 \to \mathbf{R}$ is continuous and $g : \mathbf{R} \to \mathbf{R}$ is defined by $g(t) = \int_t^{t^2} (\int_0^{x^3} f(x,y)\,dy)\,dx$. Determine $g'(t) = ?$ 10%

2. (i) Is the integral $\int_{0+}^{1} \sin\frac{1}{x}\,dx$ convergent ? Justify your answer. 10%

   (ii) Let $a_n = \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot 2n}$. Is $\{a_n\}$ convergent ? Justify your answer. 10%

3. (i) Find the work done by the force $\mathrm{F}(x,y,z) = (yz, xz, xy)$ in moving an object from $(0,0,0)$ to $(1,2,3)$ along the curve $\overrightarrow{\gamma}(t) = (t, 2t, 3t)$. 10%

   (ii) Use Green's Theorem to evaluate $\int_C x^2y\,dx + 3xy\,dy$, where C is the positively oriented simple closed curve determined by the graphs of $y = x^2$ and $y = \sqrt{x}$. 10%

4. Let f be differentiable for $x > 0$. Prove or disprove

   (i) If $\lim_{x\to\infty} f(x) = 0$, then $\lim_{x\to\infty} f'(x) = 0$. 10%

   (ii) If $\lim_{x\to 0^+} f(x) = \infty$, then $\lim_{x\to 0^+} f'(x) = -\infty$. 10%

5. (i) Evaluate $\int_0^1 (\int_x^1 \frac{\sin y}{y}\,dy)\,dx$. 10%

   (ii) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+(\tan x)^{\sqrt{3}}}$. 10%